

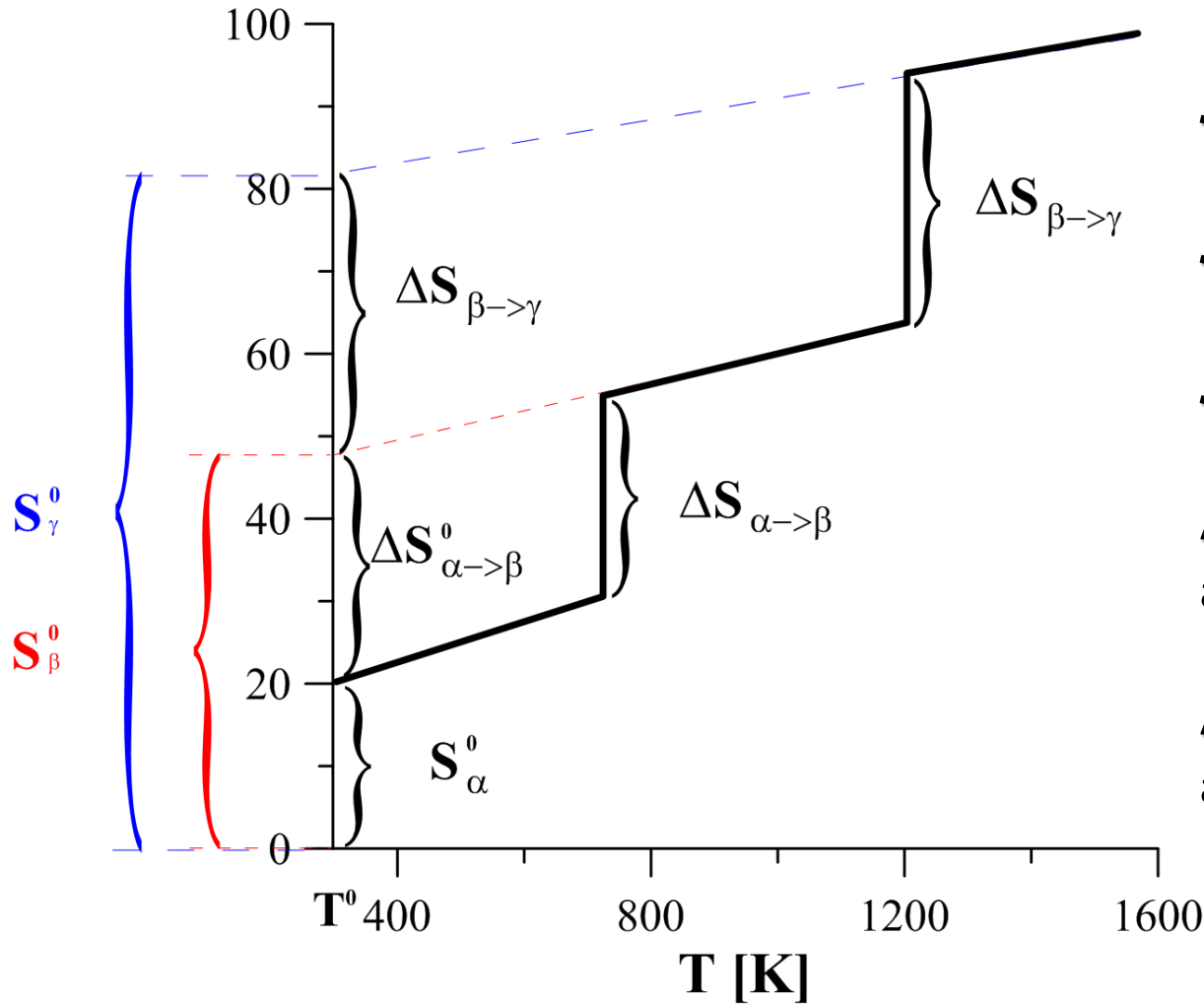
H_{α}^0 - the standard enthalpy of α phase

H_{β}^0 - the standard enthalpy of β phase

H_{γ}^0 - the standard enthalpy of γ phase

$\Delta H_{\alpha \rightarrow \beta}$ - the transition enthalpy of α to β phase at T_{t1}

$\Delta H_{\beta \rightarrow \gamma}$ - the transition enthalpy of β to γ phase at T_{t2}



$S_{\alpha}^0 = S_{X(\alpha)}^0$ - the standard entropy of α phase

$S_{\beta}^0 = S_{X(\beta)}^0$ - the standard entropy of β phase

$S_{\gamma}^0 = S_{X(\gamma)}^0$ - the standard entropy of γ phase

$\Delta S_{\alpha \rightarrow \beta} = \Delta S_{(\alpha \rightarrow \beta)}$ - the transition entropy of α to β phase at T_{t1}

$\Delta S_{\beta \rightarrow \gamma} = \Delta S_{(\beta \rightarrow \gamma)}$ - the transition entropy of β to γ phase at T_{t2}



Exercise 1

Calculate the change in the enthalpy and entropy of solid and liquid zinc in the temperature range from $T^0 = 298.16$ K to $T = 1000$ K, step = 100 K based on following data:

MPT=692.655K BPT= 1180K

s -	4.956	2.99E-3	.199E+5	0.	298.14	$H_s^0 = 0.$	$S_s^0 = 9.95$
l -	7.5	3*0.			692.655	$\Delta H_{s \rightarrow l} = 1750.$	
g -	4.968	3*0.			1180.	$\Delta H_{l \rightarrow g} = 27569.$	

Calculation of the standard enthalpy and entropy of the liquid phase $H_{lT_0}^0$ and $S_{lT_0}^0$

Step I. *Establishing the equations for C_p and enthalpis and entropies of solid, liquid and gas Zn*

$$C_{p(s)} = a + b \cdot T + c \cdot T^{-2} + d \cdot T^2$$

$$C_{p(s)} = 4.956 + 2.99E-3 \cdot T + .199E+5 \cdot T^{-2} + 0 \cdot T^2$$

$$C_{p(s)} = 4.956 + 2.99 \cdot 10^{-3} \cdot T + 0.199 \cdot 10^5 \cdot T^{-2}$$

$$C_{p(l)} = 7.5 \quad C_{p(g)} = 4.968, \quad (b=0, \quad c=0, \quad d=0)$$

$$H_{(s)}^0 = 0 \quad \Delta H_{s \rightarrow l} = 1750 \quad \Delta H_{l \rightarrow g} = 27569 \quad S_s^0 = 9.95$$

$$\Delta S_{s \rightarrow l} = \frac{\Delta H_{s \rightarrow l}}{T_{t1}} = \frac{1750}{692.655} = 2.5265 \quad \Delta S_{l \rightarrow g} = \frac{\Delta H_{l \rightarrow g}}{T_{t2}} = \frac{27569}{1180} = 23.3636$$



Step 2. *Calculation of the standard enthalpy of liquid and gas phase for Zn*

$$H_{(l)}^0 = H_{(s)}^0 + \int_{T^0}^{T_m} C_{p(s)} dT + \Delta H_{(s \rightarrow l)} - \int_{T_m}^{T^0} C_{p(l)} dT$$

$$H_{(g)}^0 = H_{(l)}^0 + \int_{T_m}^{T_b} C_{p(l)} dT + \Delta H_{(l \rightarrow g)} - \int_{T_b}^{T^0} C_{p(g)} dT$$

$$H_{(l)}^0 = 0 + \int_{T^0}^{T_m} (4.956 + 2.99 \cdot 10^{-3} \cdot T + 0.199 \cdot 10^5 \cdot T^{-2}) dT + 1750 - \int_{T_m}^{T^0} 7.5 dT$$

$$H_{(g)}^0 = H_{(l)}^0 + \int_{T_m}^{T_b} 7.5 dT + \Delta H_{(l \rightarrow g)} - \int_{T_b}^{T^0} 4.968 dT$$



Step 2. *Calculation of the standard entropy of liquid and gas phase for Zn*

$$S_{(l)}^0 = S_{(s)}^0 + \int_{T^0}^{T_m} \frac{C_{p(s)}}{T} dT + \Delta S_{(s \rightarrow l)} - \int_{T_m}^{T^0} \frac{C_{p(l)}}{T} dT$$

$$S_{(g)}^0 = S_{(l)}^0 + \int_{T_m}^{T_b} \frac{C_{p(l)}}{T} dT + \Delta S_{(l \rightarrow g)} - \int_{T_b}^{T^0} \frac{C_{p(g)}}{T} dT$$

$$S_{(l)}^0 = 9.95 + \int_{T^0}^{T_m} \frac{(4.956 + 2.99 \cdot 10^{-3} \cdot T + 0.199 \cdot 10^5 \cdot T^{-2})}{T} dT + \frac{1750}{692.655} - \int_{T_m}^{T^0} \frac{7.5}{T} dT$$

$$S_{(g)}^0 = S_{(l)}^0 + \int_{T_m}^{T_b} \frac{7.5}{T} dT + \Delta S_{(l \rightarrow g)} - \int_{T_b}^{T^0} \frac{4.968}{T} dT$$



Calculated the standard entropy and enthalpy of liquid and gas Zn

$$H_{(l)}^0 = 1368.8 \text{ [cal} \cdot \text{mol}^{-1}\text{]}$$

$$H_{(g)}^0 = 31170.65 \text{ [cal} \cdot \text{mol}^{-1}\text{]}$$

$$S_{(l)}^0 = 11.603 \text{ [cal} \cdot \text{K}^{-1} \cdot \text{mol}^{-1}\text{]}$$

$$S_{(g)}^0 = 38.450 \text{ [cal} \cdot \text{K}^{-1} \cdot \text{mol}^{-1}\text{]}$$



Step III. Calculation of the enthalpy and entropy of solid, liquid and gas Zn

$$H_{Zn(l)} = H_{Zn(l)}^0 + a_{(l)}(T - T^0) = 1368.8 + 7.5 \cdot (T - 298.16)$$

$$H_{Zn(g)} = H_{Zn(g)}^0 + a_{(g)}(T - T^0) = 31170.65 + 4.968 \cdot (T - 298.16)$$

$$S_{Zn(l)} = S_{Zn(l)}^0 + a_{(l)} \cdot \ln\left(\frac{T}{T^0}\right) = 11.603 + 7.5 \cdot \ln\left(\frac{T}{298.16}\right)$$

$$S_{Zn(g)} = S_{Zn(g)}^0 + a_{(g)} \ln\left(\frac{T}{T^0}\right) = 38.450 + 4.968 \cdot \ln\left(\frac{T}{298.16}\right)$$

Step III. Calculation of the enthalpy and entropy of solid, liquid and gas Zn

$$H_{Zn(s)} = H_{Zn(s)}^0 + a_{(s)} \cdot (T - T^0) + b_{Zn(s)} \cdot \frac{T^2 - T^{02}}{2} + c_{Zn(s)} \cdot \frac{T - T^0}{T \cdot T^0}$$

$$H_{Zn(s)} = 0 + 4.956 \cdot (T - 298.16) + 0.00299 \cdot \frac{T^2 - 298.16^2}{2} + 19900 \cdot \frac{T - 298.16}{T \cdot T^0}$$

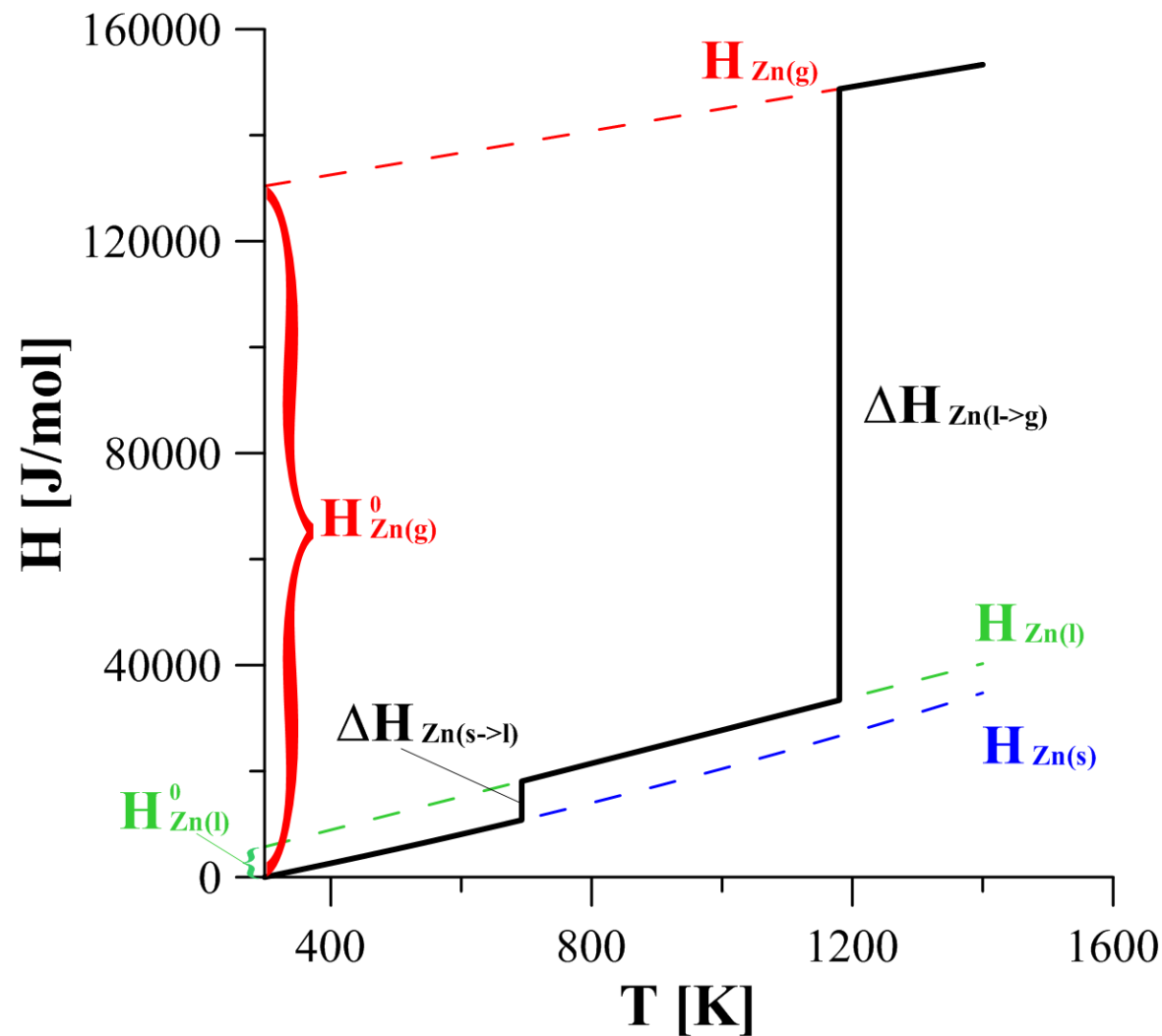
$$S_{Zn(s)} = a_{(s)} \cdot \ln\left(\frac{T}{T^0}\right) + b_{(s)} \cdot b \cdot (T - T^0) + c_{(s)} \cdot \frac{T^2 - T^{02}}{2 \cdot T^2 \cdot T^{02}}$$

$$S_{Zn(s)} = 4.956 \cdot \ln\left(\frac{T}{298.16}\right) + 0.00299 \cdot (T - T^0) + 19900 \cdot \frac{T^2 - T^{02}}{2 \cdot T^2 \cdot T^{02}}$$



T(K)	H_(s)	H_(l)	H_(g)
298.2	0	5727	130418
400	2628	8923	132535
450	3954	10492	133574
500	5306	12061	134614
550	6687	13630	135653
600	8096	15199	136692
650	9534	16768	137732
692.7	10785	18107	138618
700	11002	18337	138771
750	12500	19906	139810
800	14029	21475	140850
900	17177	24613	142928
1000	20449	27751	145007
1100	23844	30889	147085
1180	26648	33400	148748
1200	27362	34027	149164
1300	31005	37165	151243
1400	34772	40303	153321

T(K)	S_(s)	S_(l)	S_(g)
298.2	41.632	48.548	160.875
400	49.208	57.769	166.982
450	52.33	61.465	169.431
500	55.179	64.771	171.621
550	57.81	67.762	173.602
600	60.262	70.492	175.410
650	62.564	73.004	177.074
692.7	64.428	74.998	178.395
700	64.740	75.329	178.615
750	66.807	77.494	180.049
800	68.780	79.520	181.39
900	72.487	83.216	183.838
1000	75.932	86.522	186.028
1100	79.167	89.513	188.01
1180	81.628	91.716	189.469
1200	82.228	92.243	189.818
1300	85.143	94.755	191.482
1400	87.934	97.08	193.022



$$H_{(s)}^0 = H_{Zn(s)}^0$$

$$H_{(l)}^0 = H_{Zn(l)}^0$$

$$H_{(g)}^0 = H_{Zn(g)}^0$$

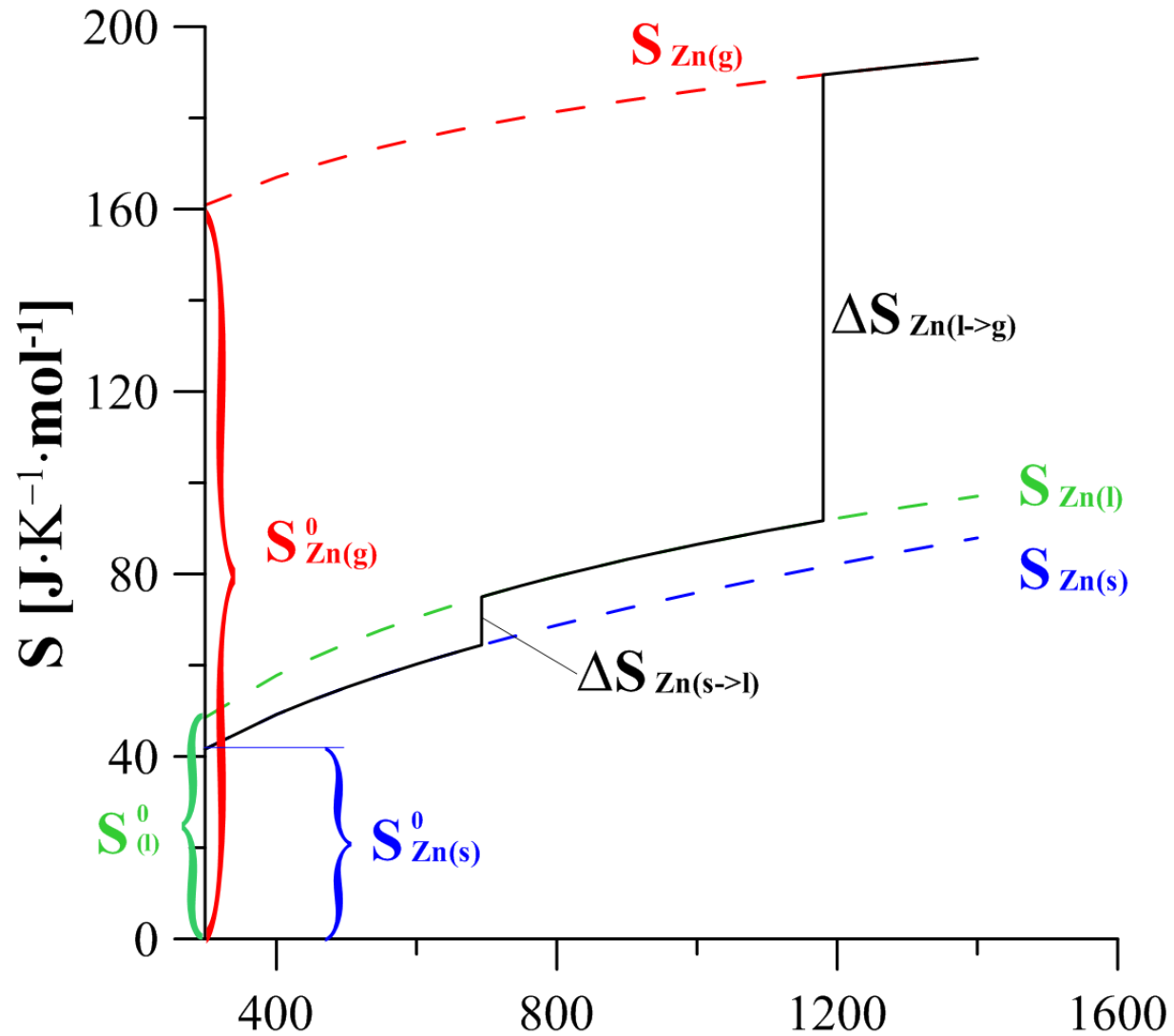
$$H_{(s)T} = H_{Zn(s)}$$

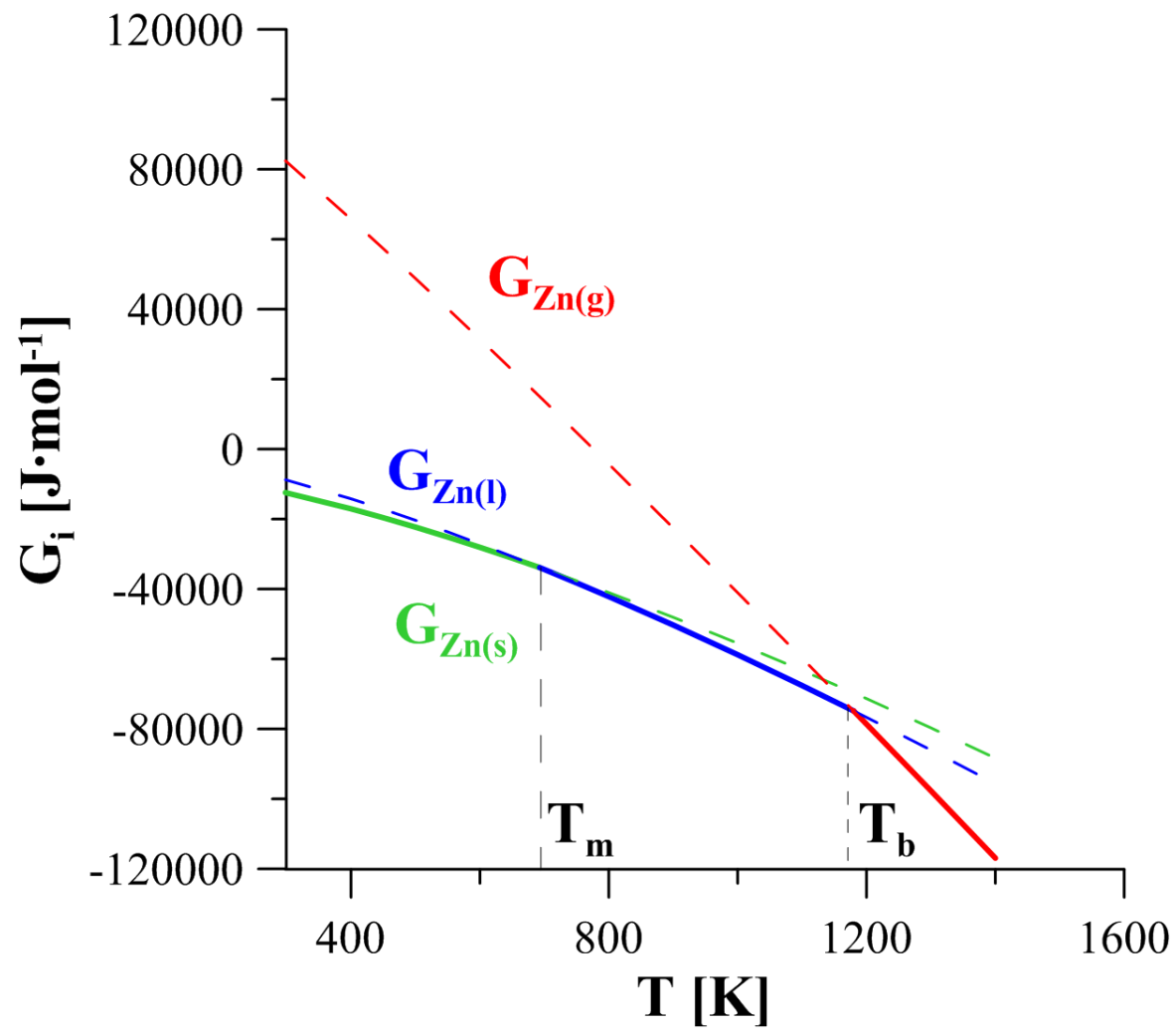
$$H_{(l)T} = H_{Zn(l)}$$

$$H_{(g)T} = H_{Zn(g)}$$

$$\Delta H_{(s \rightarrow l)T_m} = \Delta H_{Zn(s \rightarrow l)}$$

$$\Delta H_{(l \rightarrow g)T_b} = \Delta H_{Zn(l \rightarrow g)}$$







The change of the Gibbs Energy of the chemical reaction



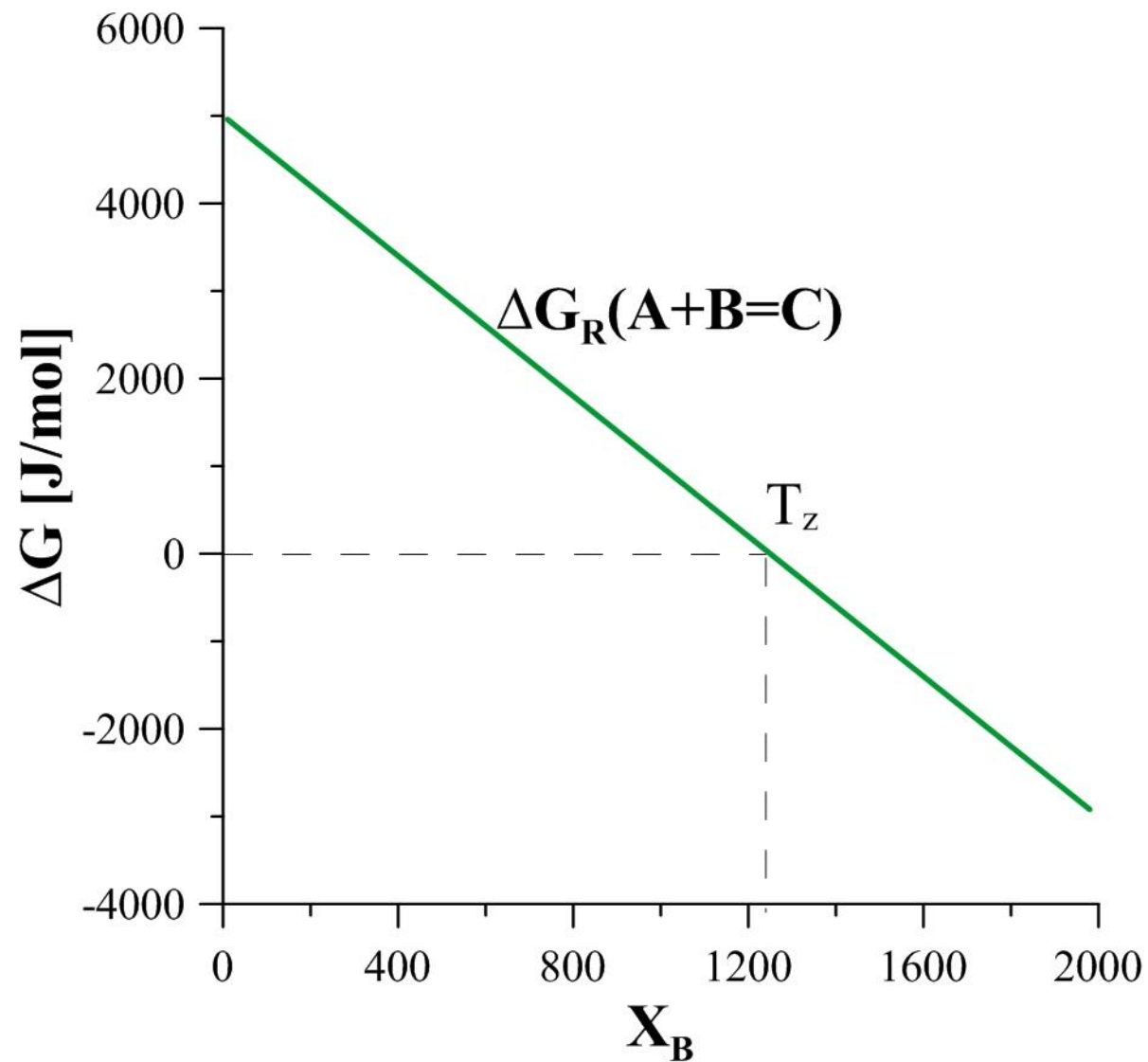
$$\Delta G_R = \Delta G_p - \Delta G_{sub}$$

$$\Delta G_R = \Delta G_C - (\Delta G_A + \Delta G_B)$$

$$\Delta G_R = \Delta G_C - \Delta G_A - \Delta G_B$$

$$G_2 - G_1 = \int_1^2 C_p dT - T \int_1^2 \frac{C_p}{T} dT$$

$$C_p = a + bT + \frac{c}{T^2} + dT^2 + \dots$$





Binary solutions

$$G_A = G_A^0 + RT \ln X_A + G_A^{ex}$$

$$G_B = G_B^0 + RT \ln X_B + G_B^{ex}$$

$$G_{AB} = X_A G_A + X_B G_B$$

$$G_{AB} = X_A (G_A^0 + RT \ln X_A + G_A^{ex}) + X_B (G_B^0 + RT \ln X_B + G_B^{ex})$$

$$X_A = 1 - X_B$$

$$X_B = 1 - X_A$$



Binary solutions - continuation

$$G_{AB}^0 = X_A G_A^0 + X_B G_B^0$$

$$\Delta G_{AB} = X_A (RT \ln X_A + G_A^{ex}) + X_B (RT \ln X_B + G_B^{ex})$$

Roztwory doskonałe

$$G_{AB}^{id} = G_{AB}^0 + X_A RT \ln X_A + X_B RT \ln X_B$$

$$\Delta G_{AB}^{id} = X_A RT \ln X_A + X_B RT \ln X_B$$



Excess Gibbs free energy

$$G_{AB}^{ex} = X_A G_A^{ex} + X_B G_B^{ex}$$

$$G_{AB}^{ex} = X_A RT \ln \gamma_A + X_B RT \ln \gamma_B$$

$$\Delta G_{AB} = X_A (RT \ln X_A + G_A^{ex}) + X_B (RT \ln X_B + G_B^{ex})$$

$$\Delta G_{AB} = X_A RT \ln X_A \gamma_A + X_B RT \ln X_B \gamma_B$$

$$a_A = X_A \gamma_A$$

$$a_B = X_B \gamma_B$$

$$\Delta G_{AB} = X_A RT \ln a_A + X_B RT \ln a_B$$



Partial Gibbs free energy of component

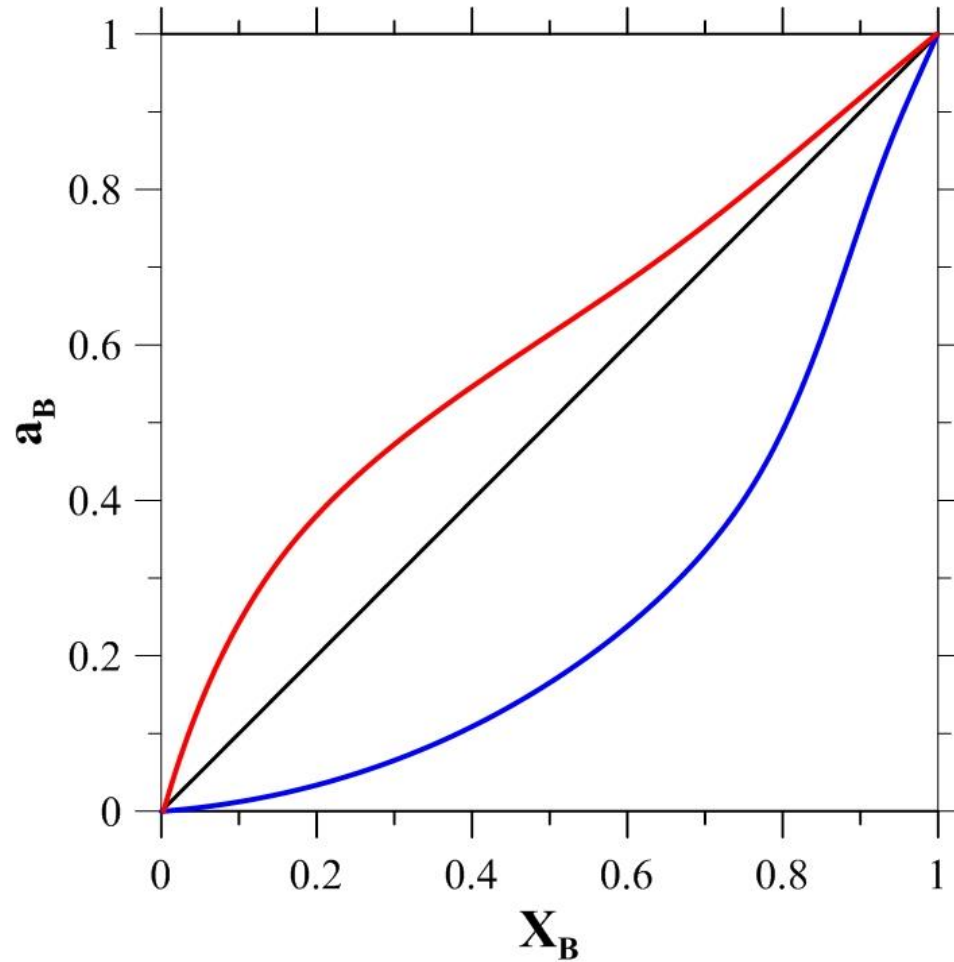
$$\Delta G_{AB} = X_A RT \ln a_A + X_B RT \ln a_B$$

$$\Delta G_A = RT \ln a_A$$

$$\Delta G_B = RT \ln a_B$$



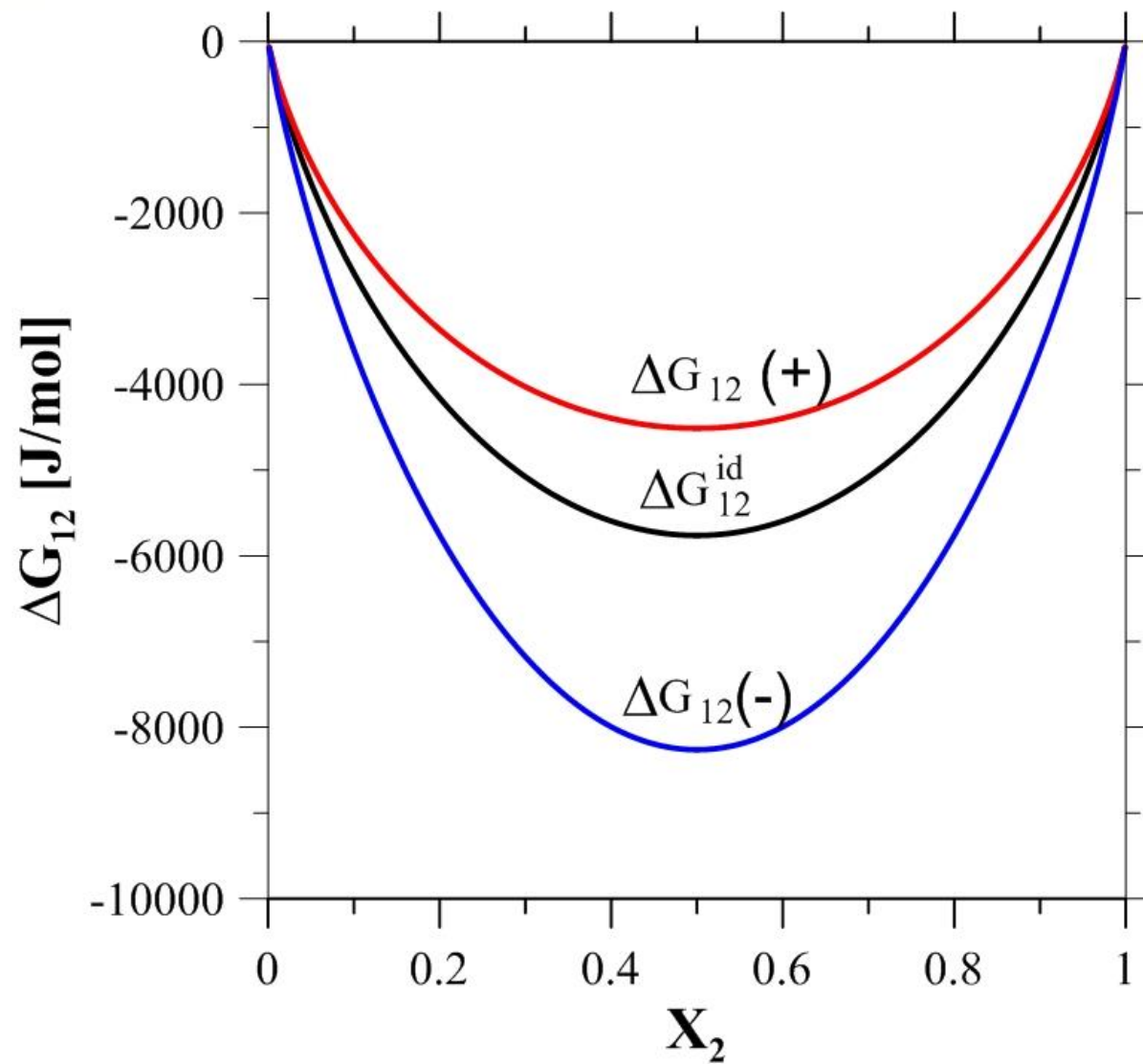
Component activity

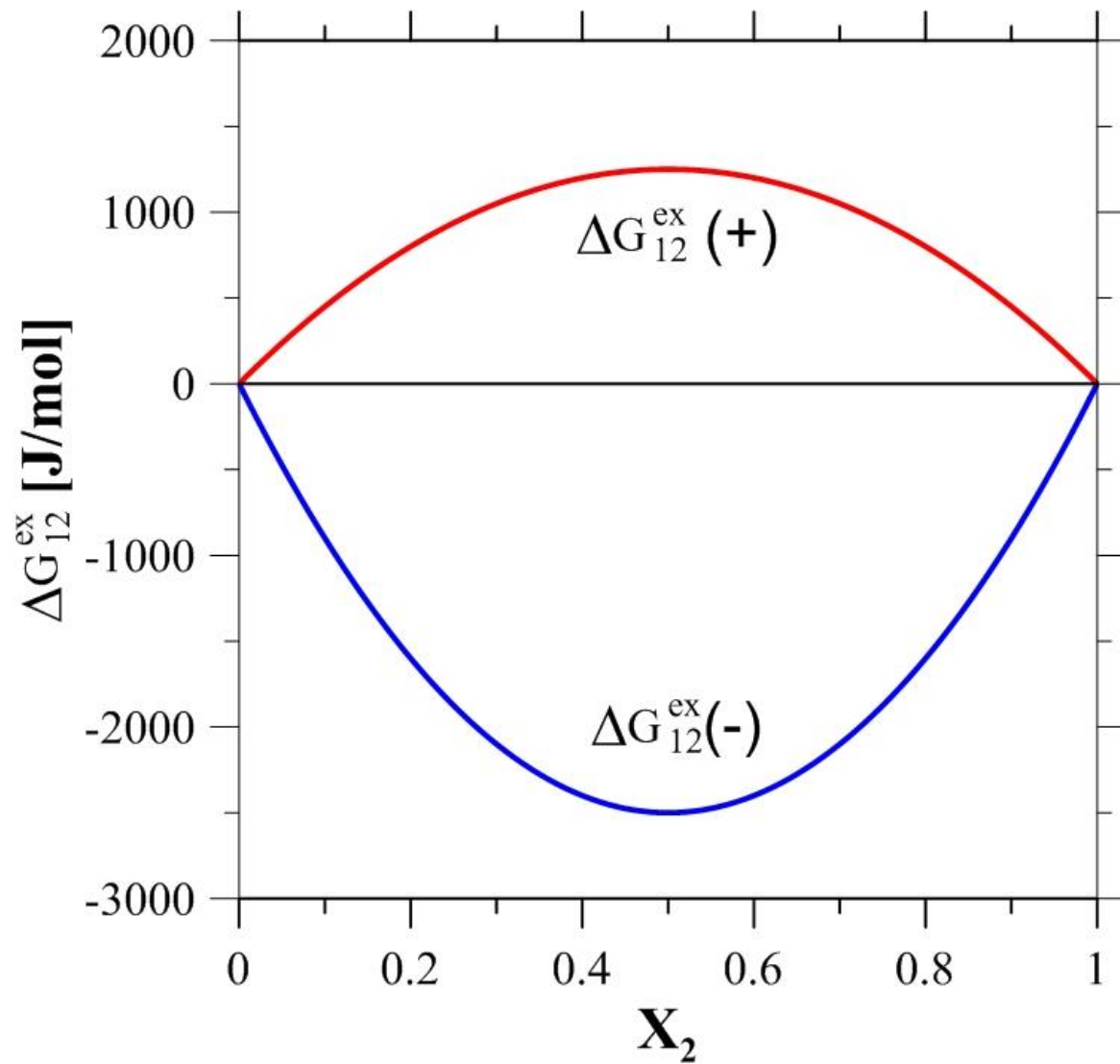


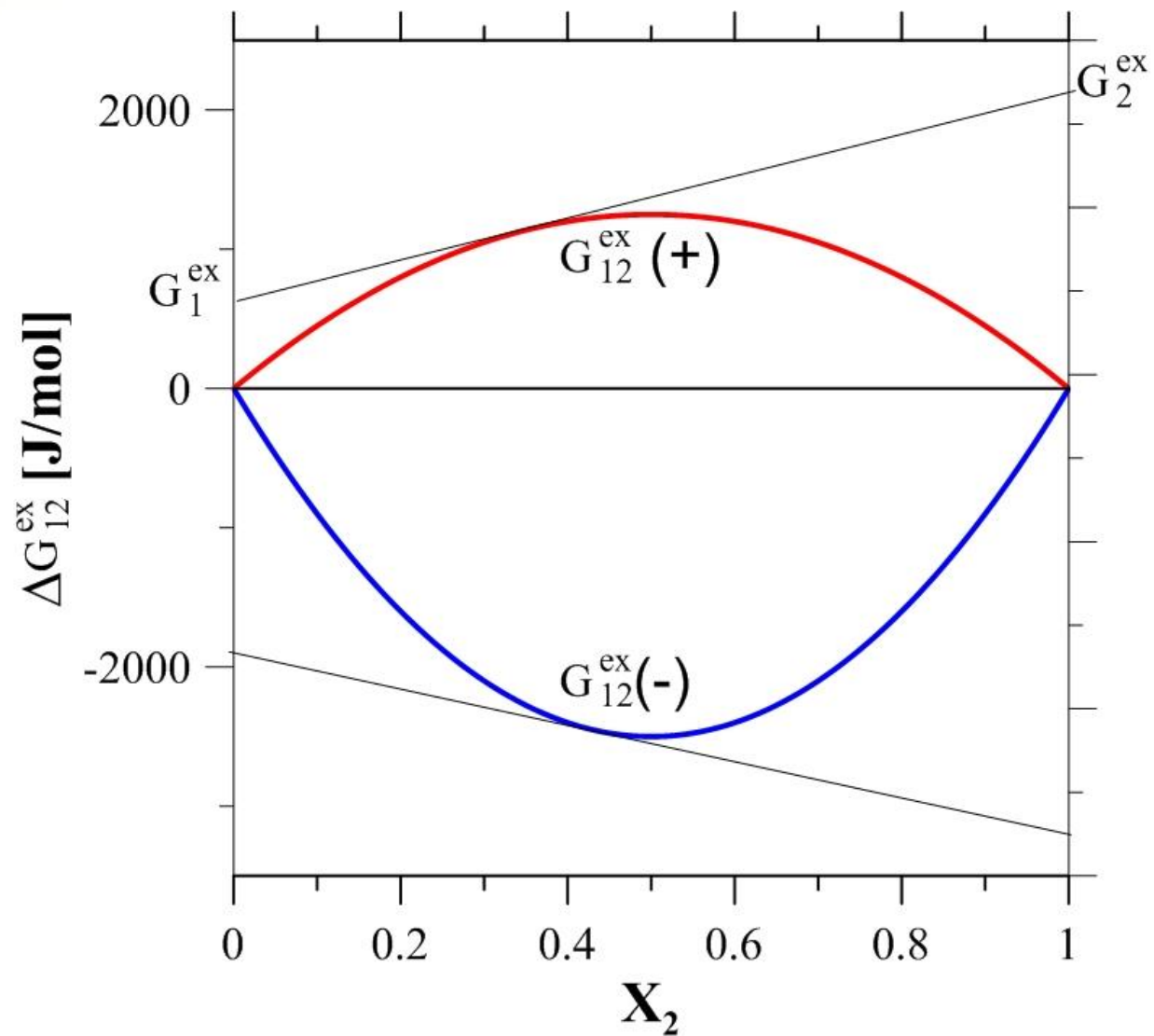
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Gibbs-Duhem equation

$$X_1 dG_1^{ex} + X_2 dG_2^{ex} = 0, \quad T = \text{const}$$

$$G_1^{ex} = G_1^{ex}(T, X_2)$$

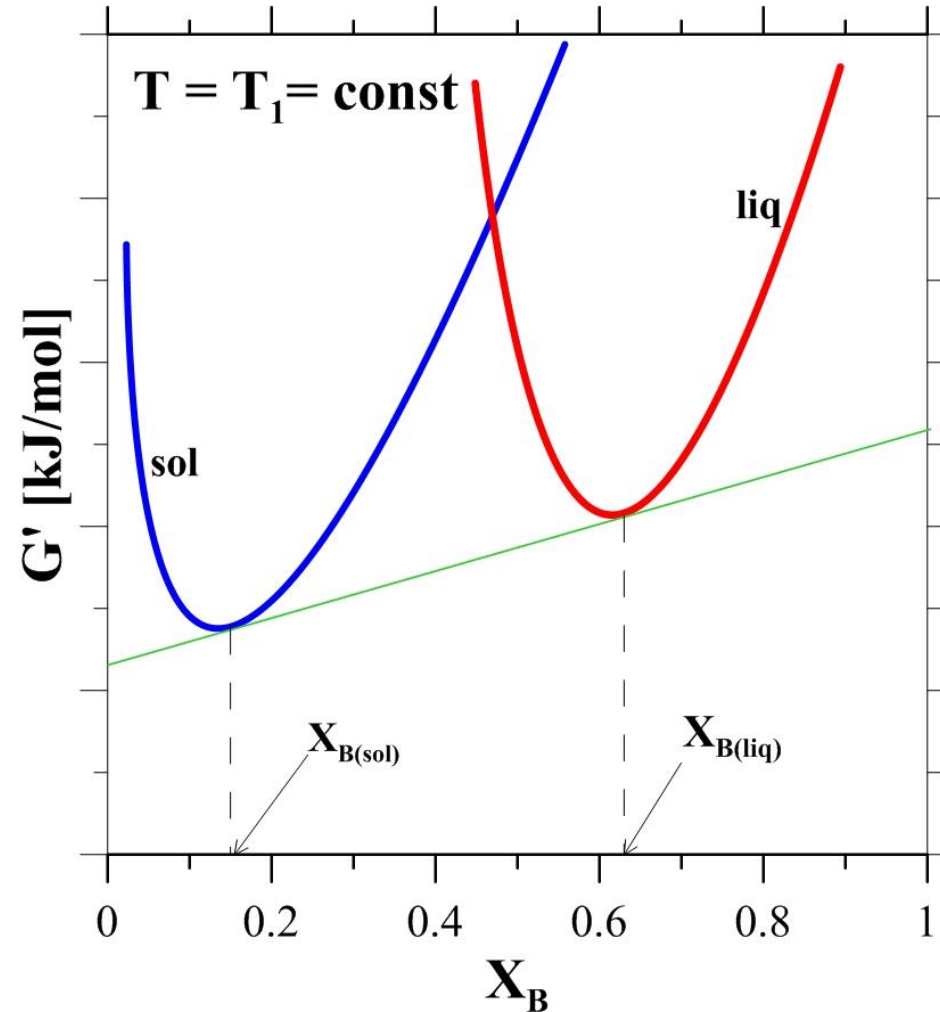
$$G_2^{ex} = G_2^{ex}(T, X_2)$$

$$dG_1^{ex} = \left(\frac{dG_1^{ex}}{dX_2} \right)_T$$

$$dG_2^{ex} = \left(\frac{dG_2^{ex}}{dX_2} \right)_T$$



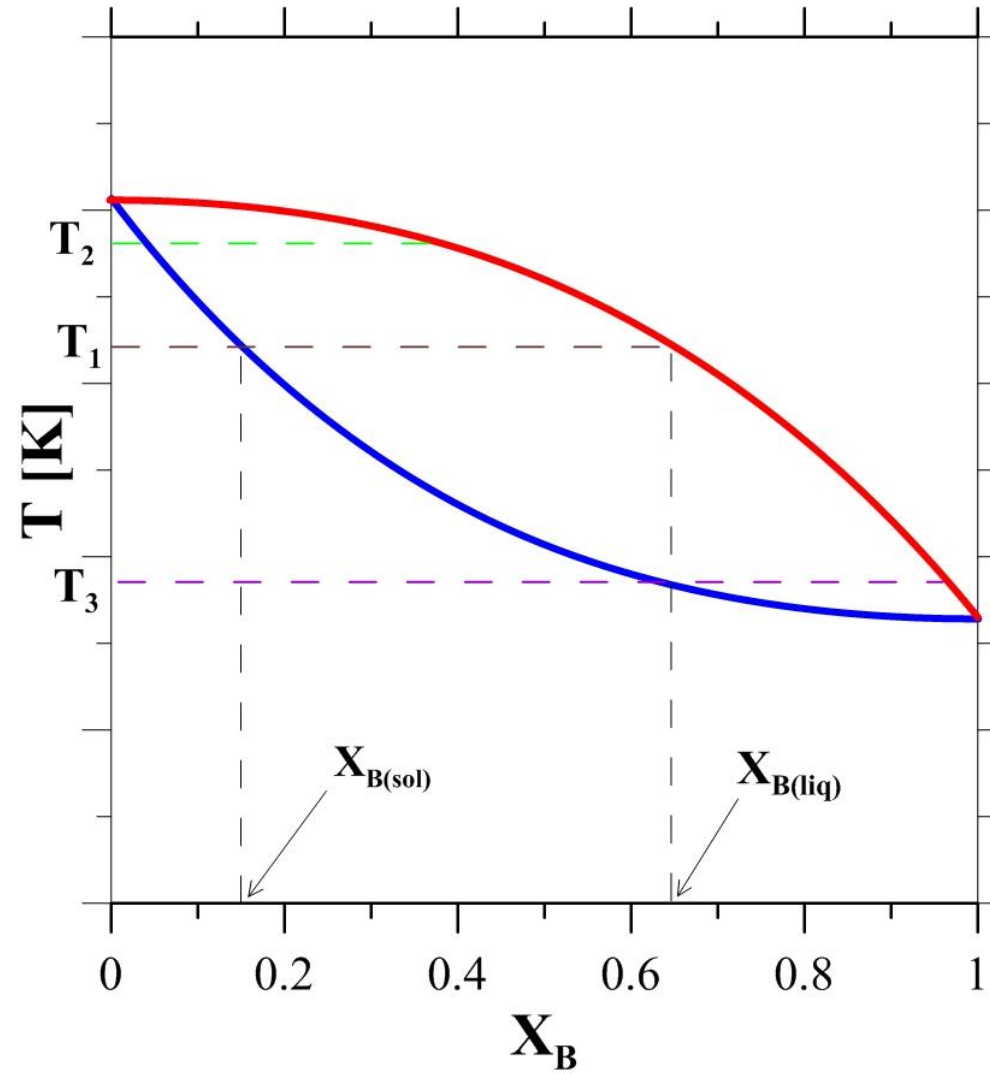
Interphase equilibria– Thermodynamic analysis

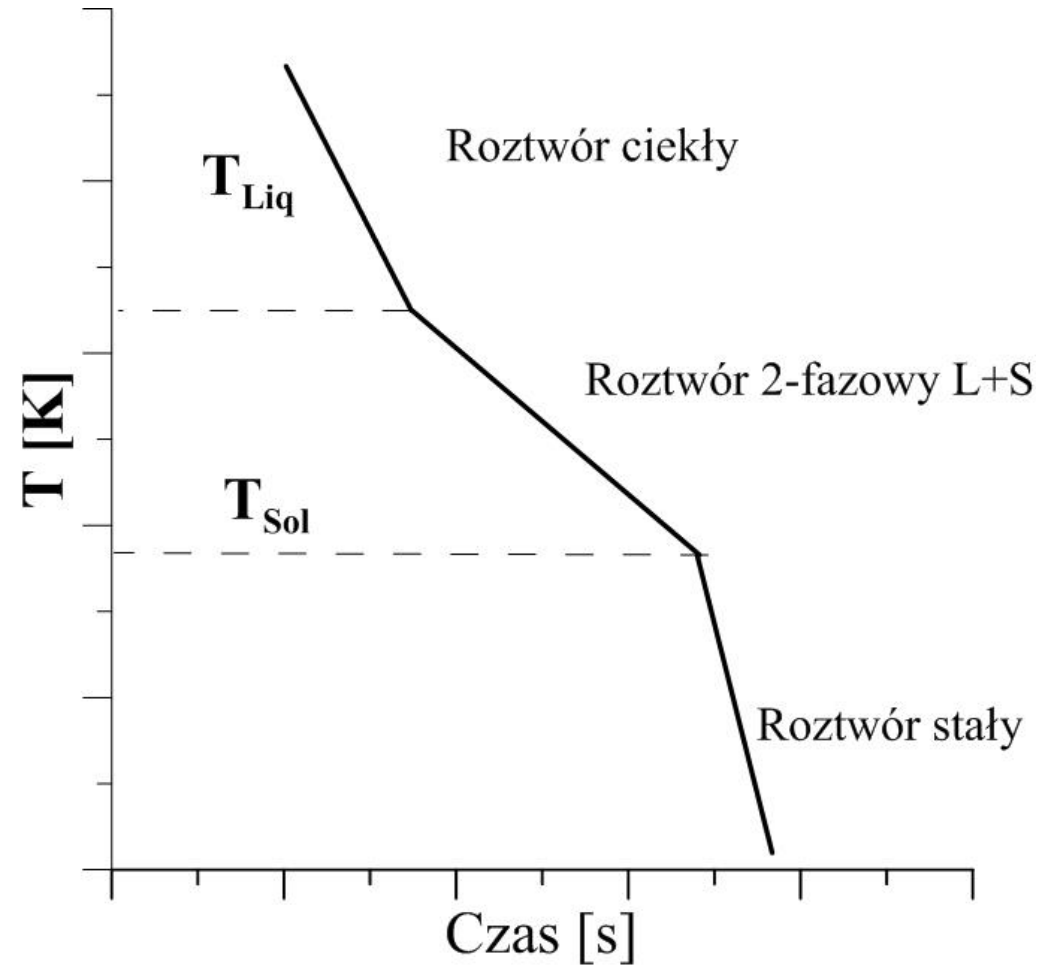
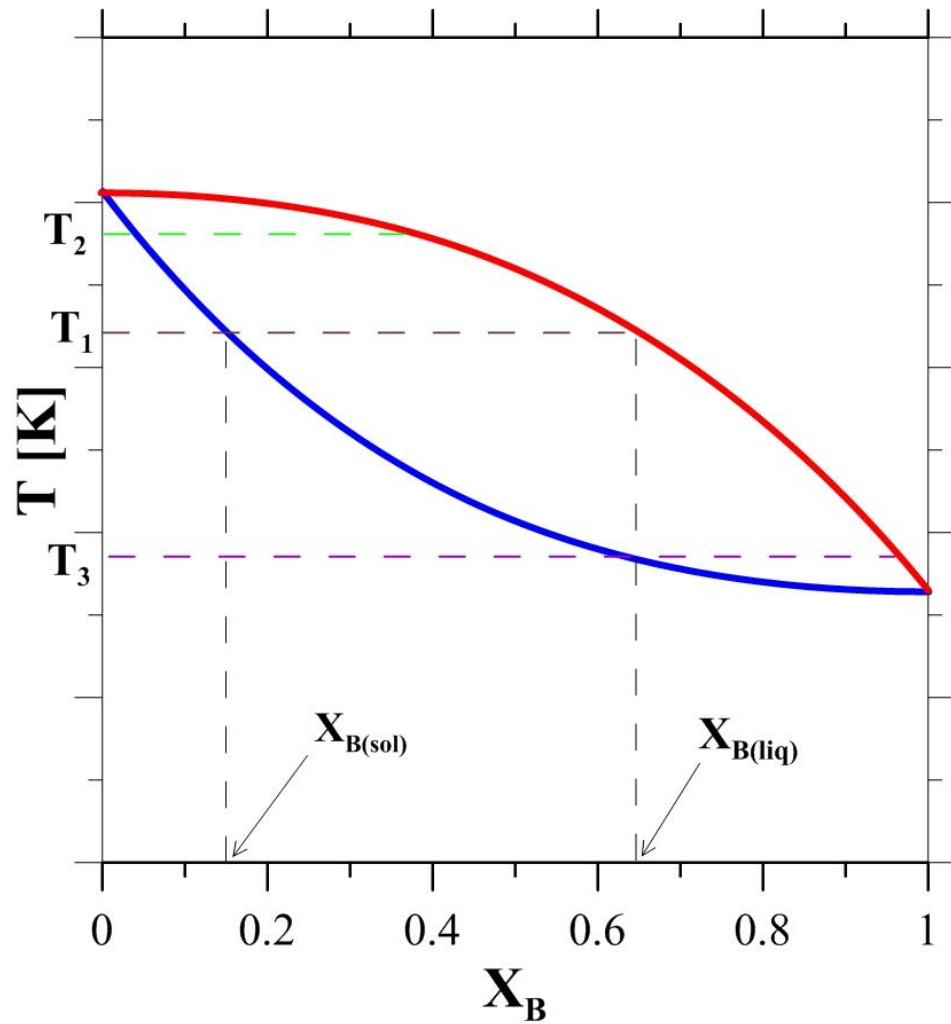


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Applying the Gibbs-Duhem equation

a) Partial function of component 2 is known

$$X_1 dG_1^{ex} + X_2 dG_2^{ex} = 0, \quad G_2^{ex} = V(X_2), \quad T = \text{const}$$

$$G_1^{ex} = G_1^{ex}(X_2) \quad G_2^{ex} = G_2^{ex}(X_2)$$

$$dG_1^{ex} = \left(\frac{dG_1^{ex}}{dX_2} \right)_T \quad dG_2^{ex} = \left(\frac{dG_2^{ex}}{dX_2} \right)_T$$

$$X_1 \frac{dG_1^{ex}}{dX_2} = -X_2 \frac{dG_2^{ex}}{dX_2}$$

$$\frac{dG_1^{ex}}{dX_2} = - \frac{X_2}{1 - X_2} \frac{dG_2^{ex}}{dX_2}$$



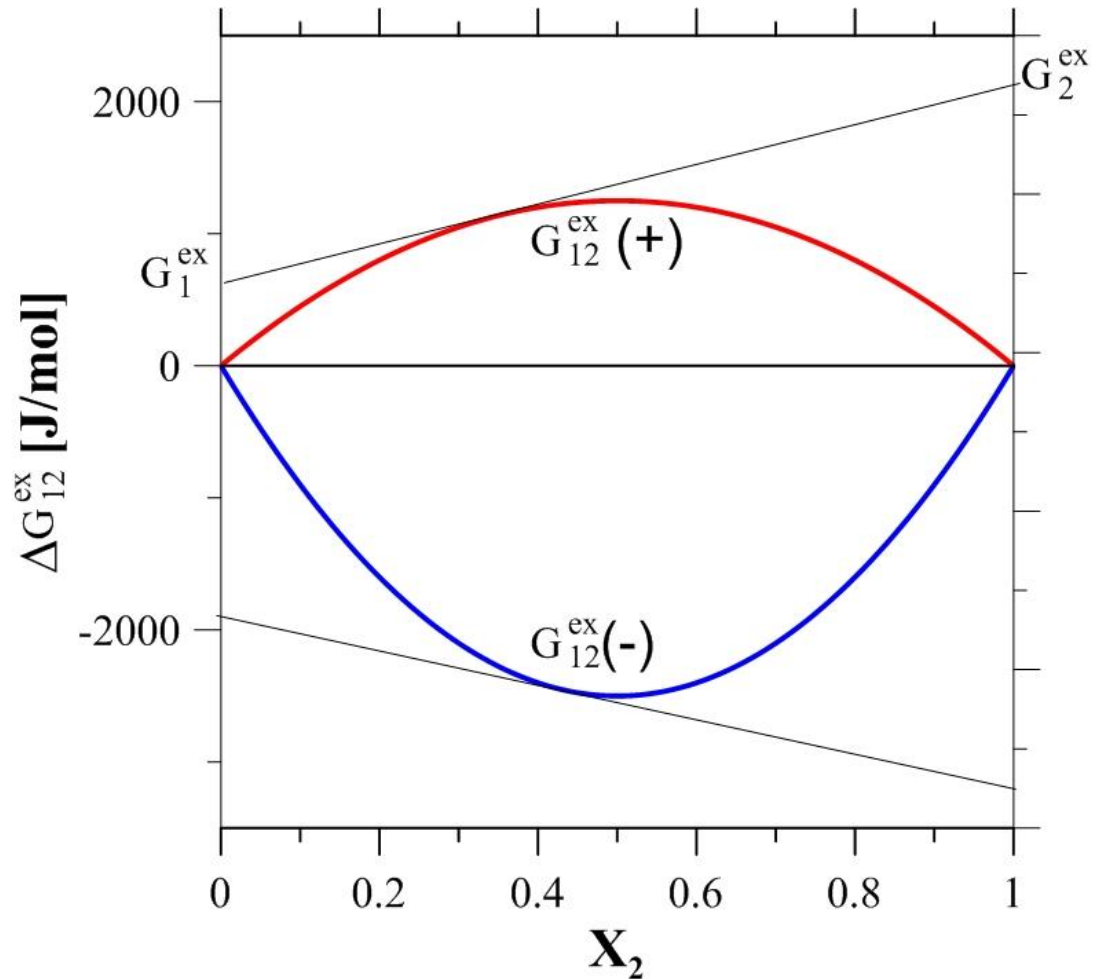
Applying the Gibbs-Duhem equation

$$\frac{dG_1^{ex}}{dX_2} = -\frac{X_2}{1-X_2} \frac{dG_2^{ex}}{dX_2}$$

$$\int \frac{dG_1^{ex}}{dX_2} dX_2 = \int -\frac{X_2}{1-X_2} \frac{dG_2^{ex}}{dX_2} dX_2$$

$$\int \frac{dG_1^{ex}}{dX_2} dX_2 = \int -\frac{X_2}{1-X_2} \frac{dV(X_2)}{dX_2} dX_2$$

$$G_1^{ex} = W(X_2) + C$$



$$G_1^{ex} = W(X_2) + C$$

To calculate C constant the G_1^{ex} value for any X_2 (X_1) value must be known.

From the figure on the left side one can notice that:

for $X_2=0$ ($X_1=1$), $G_1^{ex} = 0$

and it gives

$$C = -W(X_2 = 0)$$



Example:

The partial excess Gibbs energy of component A in binary liquid alloys A-B is given by following equation:

$$G_B^{ex} = 2500(1 - X_B)^2 \quad T = \text{const.}$$

Calculate the excess Gibbs Energy the component B.

$$\frac{dG_B^{ex}}{dX_B} = 5000(1 - X_B)$$

From the Gibbs-Duhem equation it is known that:

$$\frac{dG_A^{ex}}{dX_B} = -\frac{X_B}{1 - X_B} \frac{dG_B^{ex}}{dX_B}$$



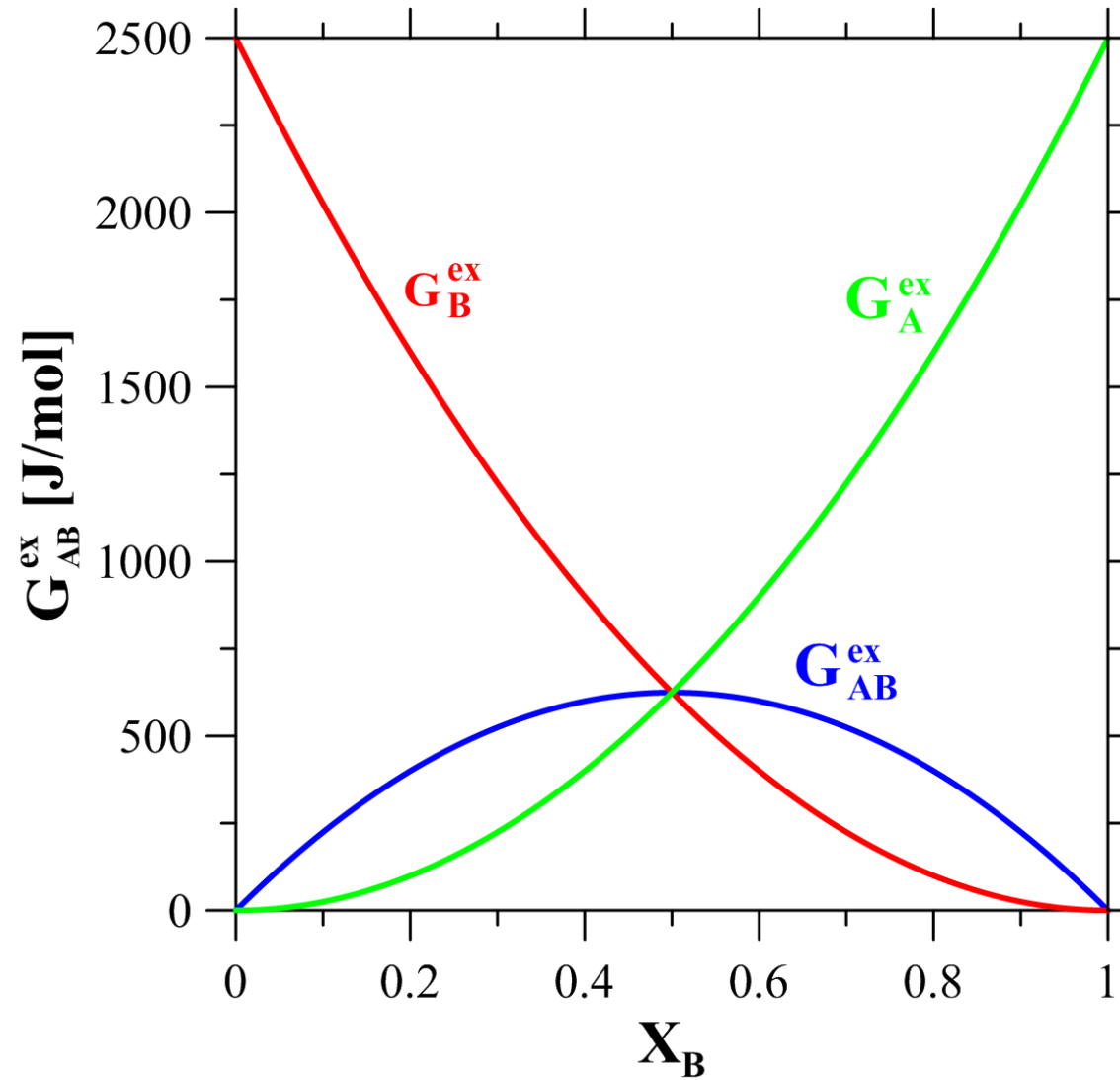
$$\int \frac{dG_A^{ex}}{dX_B} dX_B = 5000 \int -\frac{X_B}{1-X_B} (1-X_B) dX_B$$

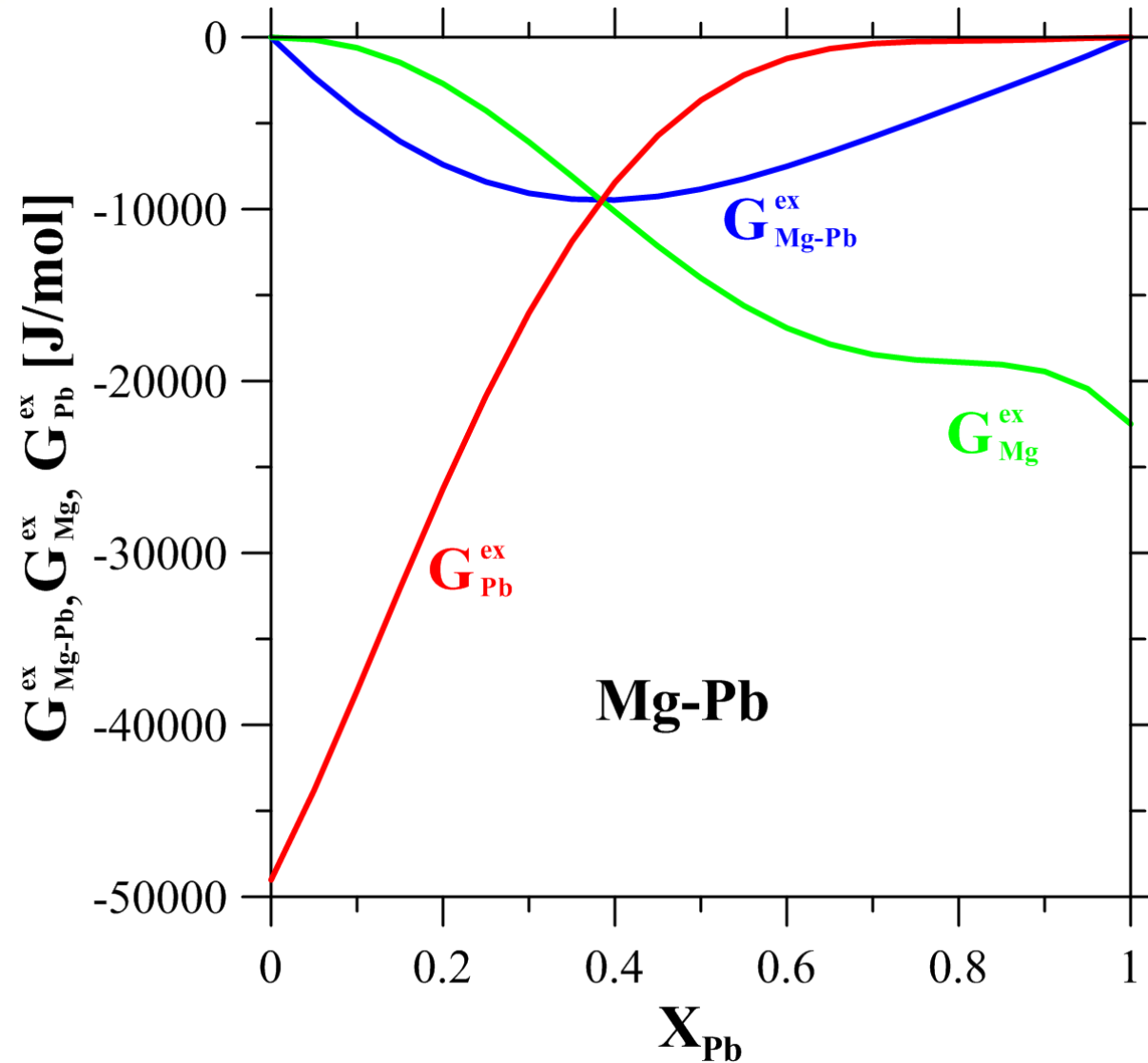
$$G_A^{ex} = 5000 \frac{X_B^2}{2} = 2500X_B^2$$

$$G_B^{ex} = 2500(1 - X_B)^2 = 2500X_A^2$$

$$G_{AB}^{ex} = X_A G_A^{ex} + X_B G_B^{ex}$$

$$G_{AB}^{ex} = 2500X_A X_B$$







Applying the Gibbs-Duhem equation

b) Molar Gibbs energy of solution (1-2) is known

$$G^{ex} = X_1 G_1^{ex} + X_2 G_2^{ex} = (1 - X_2) G_1^{ex} + X_2 G_2^{ex}$$

$$\frac{dG^{ex}}{dX_2} = d(G_1^{ex} + X_2 G_1^{ex}) + X_2 G_2^{ex}$$

$$\frac{dG^{ex}}{dX_2} = \frac{dG_1}{dX_2} - \left(G_1^{ex} + X_2 \frac{dG_1^{ex}}{dX_2} \right) + G_2^{ex} + X_2 \frac{dG_2^{ex}}{dX_2}$$

$$\frac{dG^{ex}}{dX_2} = \frac{dG_1}{dX_2} - G_1^{ex} - X_2 \frac{dG_1^{ex}}{dX_2} + G_2^{ex} + X_2 \frac{dG_2^{ex}}{dX_2}$$



$$\frac{dG^{ex}}{dX_2} = \frac{dG_1^{ex}}{dX_2} - G_1^{ex} - X_2 \frac{dG_1^{ex}}{dX_2} + G_2^{ex} + X_2 \frac{dG_2^{ex}}{dX_2}$$

$$\frac{dG^{ex}}{dX_2} = \frac{dG_1}{dX_2} - G_1^{ex} - (1 - X_1) \frac{dG_1^{ex}}{dX_2} + G_2^{ex} + X_2 \frac{dG_2^{ex}}{dX_2}$$

$$\frac{dG^{ex}}{dX_2} = \frac{dG_1}{dX_2} - G_1^{ex} + G_2^{ex} - (1 - X_1) \frac{dG_1^{ex}}{dX_2} + X_2 \frac{dG_2^{ex}}{dX_2}$$

$$\frac{dG^{ex}}{dX_2} = \frac{dG_1}{dX_2} + G_2^{ex} - G_1^{ex} - \frac{dG_1^{ex}}{dX_2} + X_1 \frac{dG_1^{ex}}{dX_2} + X_2 \frac{dG_2^{ex}}{dX_2}$$

$$\frac{dG^{ex}}{dX_2} = G_2^{ex} - G_1^{ex} / * X_2$$



$$X_2 \frac{dG^{ex}}{dX_2} = X_2 G_2^{ex} - X_2 G_1^{ex}$$

$$X_2 \frac{dG^{ex}}{dX_2} = X_2 G_2^{ex} - (1 - X_1) G_1^{ex}$$

$$X_2 \frac{dG^{ex}}{dX_2} = X_2 G_2^{ex} - G_1^{ex} + X_1 G_1^{ex}$$

$$G_1^{ex} = G^{ex} - X_2 \frac{dG^{ex}}{dX_2}$$

$$G_2^{ex} = G^{ex} + X_1 \frac{dG^{ex}}{dX_1} = G^{ex} + (1 - X_1) \frac{dG^{ex}}{dX_2}$$



Measurement of the thermodynamic properties

$$G_{AB} = X_A(G_A^0 + RT\ln X_A + G_A^{ex}) + X_B(G_B^0 + RT\ln X_B + G_B^{ex})$$

$$G_{AB} = X_A G_A^0 + X_B G_B^0 + X_A RT\ln X_A + X_B RT\ln X_B + X_A G_A^{ex} + X_B G_B^{ex}$$

$$G_{AB}^{id} = [X_A G_A^0 + X_B G_B^0 + X_A RT\ln X_A + X_B RT\ln X_B]$$

$$G_{AB}^{ex} = X_A G_A^{ex} + X_B G_B^{ex}$$

$$G = H - TS$$

$$G_i = H_i - TS_i$$



Electromotive force measurements of activity (Au-Li)



$$\Delta G_{Li} = -nFE = RT \ln(X_{Li})$$

$$a_{Li} = \exp\left(\frac{-n \cdot F \cdot E}{R \cdot T}\right)$$

$$G_{Li}^E = \Delta G_{Li} - RT \ln(X_{Li}) = -nFE - RT \ln(X_{Li})$$

$$\Delta S_{Li} = -\left(\frac{d\Delta G_{Li}}{dT}\right)_p = nF \left(\frac{dE}{dT}\right)_p = nFB$$



$$\Delta H_{Li} = \Delta G_{Li} + T\Delta S_{Li} = -nF \left[E - nF \left(\frac{dE}{dT} \right)_p \right] = -nFA$$

$$S_{Li}^E = \Delta S_{Li} + R \ln(X_{Li})$$

where: F – Faraday constant, R – universal gas constant, T – absolute temperature [K],
 E – electromotive force as a function T , X_{Li} – concentration of lithium in the alloy, a_{Li} – activity of Li
 ΔG_{Li} – change in the partial free enthalpy of lithium, ΔS_{Li} – change in the partial entropy of lithium,
 ΔH_{Li} – change in the partial enthalpy of lithium, S_{Li}^E – partial excess entropy of lithium.



The calorimetric study of the mixing enthalpy

$$H_{\text{DISS-X}} = (\Delta H_{\text{Signal}} \cdot K) - (H_X^{\text{T}_R \rightarrow \text{T}_M} \cdot n_X)$$

$$\Delta H_{\text{mix}} = \frac{\sum H_{\text{DISS-X}}}{n_{\text{Au}} + n_{\text{Li}} + n_{\text{Sn}}}$$

where ΔH_{Signal} is the heat effect measured after each drop of metal samples (Au, Li or Sn) into the metallic bath (Au, Li, Au-Li or Au-Sn), K is the calibration constant, T_R and T_M are the drop temperature (room temperature) and the calorimeter temperature of the respective measurement in Kelvin.

$H_X^{\text{T}_R \rightarrow \text{T}_M}$ is the enthalpy change of the pure metals (Au, Li or Sn) from the room temperature (T_R) to the measurement temperature (T_M), n_{Au} , n_{Li} and n_{Sn} are the numbers of moles of gold, lithium and tin, respectively. $H_{\text{DISS-X}}$ is the enthalpy of dissolution of pure gold, lithium or tin.



Direct reaction method



where: T_R is the temperature of the components before the introduction into the reaction zone (crucible), T is the temperature at which the reaction goes on (crucible), and Q is the heat effect measured by the calorimeter. In such a case, the formation enthalpy $\Delta_f H$ at temperature T is calculated according to the following equation:

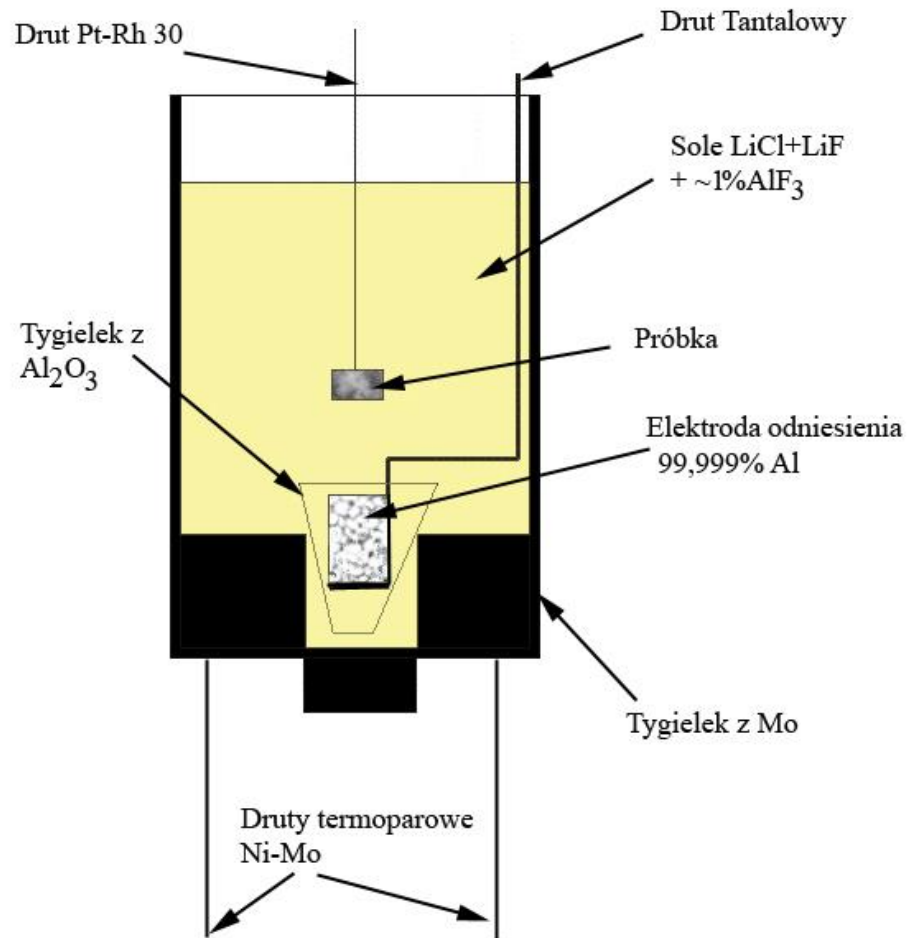
$$\Delta_f H = Q - (X_A \Delta H_A + X_B \Delta H_B)$$

Where: X_A and X_B are the mole fractions of the reacted components (A, B) and ΔH_A and ΔH_B are the enthalpy changes of the solid metals (Au or Li) between temperatures T_R and T .



Task to do:

Korzystając z definicji funkcji mola roztworu funkcji namiarowej oraz równania Gibbsa –Duhema wyprowadzić równanie dla nadmiarowej energii swobodnej Gibbsa składnika 2 – pisemnie.





Entalpię tworzenia faz w układach trójskładnikowych określa się z wykorzystaniem zależności:

1. Tygiel alundowy.
2. Stos termoparowy NiAl-NiCr.
3. Mieszadło.
4. Doprowadzenie próbek do kąpielii z temperatury pokojowej.
5. Termopara mierząca temperaturę w tyglu.
6. Korpus bloku kalorymetrycznego.
7. Pojemnik ze stali żaroodpornej.
8. Piec kalorymetru.
9. Pojemnik pośredni.
10. Doprowadzenie próbek do pojemnika pośredniego.
11. Termopara mierząca temperaturę w pojemniku pośrednim.

Schemat bloku kalorymetru typu rozpuszczania

$$\Delta_f \mathbf{H} = X_A \Delta H_A^{ef} + X_B \Delta H_B^{ef} + X_C \Delta H_C^{ef} - \Delta H_{A_{X_A} B_{X_B} C_{X_C}}^{ef}$$

