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A NEW INVERTED TORSION PENDULUM-BASED MECHANICAL SPECTROMETER TO STUDY SOFT MATTER

A new multifunctional mechanical spectrometer is developed based on an inverted torsion pendulum showing high precision in a wide frequency range to study soft matter. Apart from measuring the internal friction of solids it can also be used to study viscoelasticity. In this report we describe basic principles of the novel instrument.

Keywords: Soft matter, mechanical spectrometer, inverted torsion pendulum, viscoelasticity, mechanical spectroscopy, viscosity

1. Introduction

Rheological approach is mainly used to study the flow and transformation of substances, including fluids and other soft matter [1] like colloids, emulsions, etc. There are three kinds of traditional rheometers measuring the viscosity of fluids. The first one is capillary rheometer where the test fluid is made to flow through a narrow tube as a result of hydrostatic or applied pressure. The capillary measurements are considered as the most precise way of determining the viscosity of Newtonian and some non-Newtonian viscous fluids. In general, it has relatively simple design and is less expensive. The second one is the falling ball rheometer. This instrument measures the viscosity of Newtonian liquids and gases. The method applies Newton's law of motion under force balance on a falling sphere ball when it reaches a terminal velocity. The last one is a rotational viscometer, which uses the idea that the torque required to turn an object in a fluid as a function of the viscosity of that fluid. It measures the torque required to rotate a disk or bob in a fluid at known speed. Rotational viscometers have various conventional geometries, among which "cup and bob" viscometers, known as either the "Couette" or "Searle" systems are distinguished by whether the cup or bob rotates. 'Cone and Plate' viscometers use a cone of very shallow angle in bare contact with a flat plate. Although many efforts have been made to improve these three kinds of traditional rheometers, a novel viscometer with a high precision is still in an urgent demand. An inverted torsion pendulum is a typical mechanical spectrometer used to study mechanical loss in solids [2]. This kind of a low-frequency mechanical spectrometer is widely used to study numerous relaxation phenomena, e.g. point defects, dislocations, phase transitions [2-4], grain boundary relaxations in metals and alloys [5-7], glass transition in various materials [2-4,8,9], etc. Recently the inverted pendulum has been proposed to study soft matter such as polymer solution [10], granular materials [11], etc. and some meaningful results have been achieved by applying the modified inverted pendulum. However there still lacks a systematic description on the modified inverted pendulum as a new mechanical spectrometer measuring the viscosity and viscoelasticity of soft matter. In this report, the detailed working principle of this novel mechanical spectrometer based on an inverted torsion pendulum is presented.

2. The principle of measuring the viscosity of liquids

By attaching a Couette-like set-up onto the conventional inverted torsion pendulum, we develop a new mechanical spectrometer for soft matter, as illustrated schematically in Fig. 1. In the absence of the liquid, a mechanical spectrometer is essentially a conventional inverted torsion pendulum and can operate in forced oscillations under an external harmonic torque produced by Helmholtz coils. The forced torsion vibration is described by:

$$I\ddot{\theta} + (k + ik')\theta = M_0 e^{i\omega t}, \quad (1)$$

where I is the moment of inertia of the torsion pendulum, θ is the vibration angle, k and k' are the restoring elastic and dissipative torque coefficients due to the twisting element, respectively. $M_0 e^{i\omega t}$ represents an external oscillation torque, where ω is the angular frequency.

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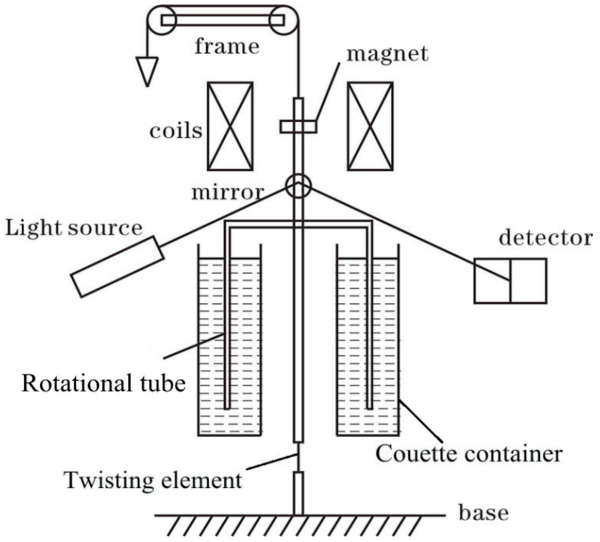

Fig. 1. Illustration of the mechanical spectrometer designed to study liquids

In its steady solution, the vibration angle θ lags behind the external torque by the phase angle δ. The phase angle δ depends on frequency:

$$\tan\delta = \frac{\tan\phi\omega_0^2}{\omega_0^2 - \omega^2}, \tag{2}$$

where $\omega_0 = \sqrt{k/I}$ is the resonant angular frequency of the system and $\tan\phi = k'/k$ represents internal friction. After filling liquid into the Couette container, the liquid inside the Couette cell imposes a damping moment M_L to the pendulum due to its viscosity. Forced torsional oscillations of mechanical spectrometer are described by

$$I\ddot{\theta} + M_L + (k + ik')\theta = M_0 e^{i\omega t}. \tag{3}$$

An external harmonic torsional moment $M_0 e^{i\omega t}$ results in the vibration angle θ, which lags behind the external torque by a new phase angle δ'

$$\theta = \theta_0 e^{i\omega t + \delta'}. \tag{4}$$

According to Landau's theory [12], we have the M_L:

$$M_L = -2Sr^2\theta_0\sqrt{\omega^3\rho\eta}\cdot e^{i\omega t+\delta'}\cdot e^{i\frac{3\pi}{4}}, \tag{5}$$

Then we obtain the following expression for $\tan\delta'$

$$\tan\delta' = \frac{k' + 2Sr^2\theta_0\sqrt{\omega^3\rho\eta}\sin\dfrac{3\pi}{4}}{k - I\omega^2 + 2Sr^2\theta_0\sqrt{\omega^3\rho\eta}\cos\dfrac{3\pi}{4}}, \tag{6}$$

where S represents the contact area between an inverted torsion pendulum and fluid, r is the radius of rotational tube, ρ denotes the fluid density, and η is the viscosity of fluid. The phase lag δ' can be determined experimentally, whereas the viscosity of fluid can be obtained from:

$$\eta = \frac{\left[\tan\delta'\left(k - I\omega^2\right) - k'\right]^2}{2S^2r^4\omega^3\rho\left(1 + \tan\delta'\right)^2}. \tag{7}$$

The error in the estimation of viscosity of a liquid, as calculated from Eq. (7) depends on:

1. The inaccuracy of the system inertial moment, the non-load resonant frequency, the elastic and dissipative spring constants of the twisting element.
2. The inaccuracy in the estimation of the area immersing in the fluid, which is directly in contact with the rotational cup and measurement error in the estimation the phase difference between the external torsional torque and the response defined as the torsion angle.

Among these two parts, the error of viscosity of the liquid is mainly caused by the last one, which can be greatly reduced by the improvement of hardware solutions, electronics and software. Because the phase difference can be precisely measured, the precision in estimation of viscosity of liquid is high. Figure 2 shows the comparison between the viscosity of distilled water measured by the novel mechanical spectrometer based on an inverted torsion pendulum and the reported value in the literature at different temperatures. It is readily shown that our developed mechanical spectrometer have a high precision.

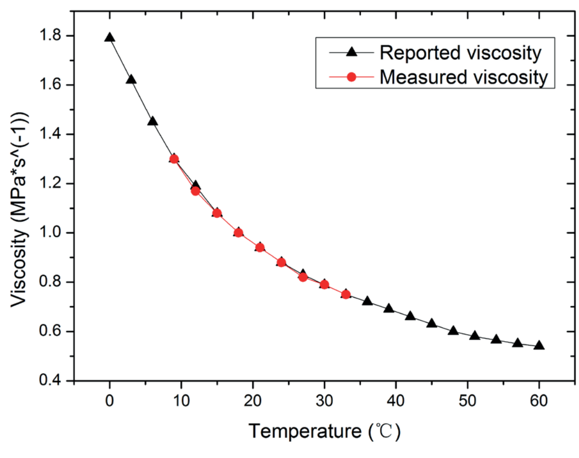

Fig. 2. Comparison of the measured viscosity and reported viscosity of distilled water at different temperatures

3. The principle of measuring the viscoelasticity of soft matter

When we fill soft matter displaying viscoelasticity instead of liquid into the Couette container, the soft matter inside the Couette cell imposes not only a damping moment M_L due to its viscosity but also the restoring moment to the pendulum as well, which is described by $(R+iX)\dot{\theta}$ [9]. The torsion vibration of the mechanical spectrometer can be described by:

$$(R+iX)\dot{\theta}+I\ddot{\theta}+(k+ik')\theta=M_0e^{i\omega t}. \quad (8)$$

In its steady solution, the apparent internal friction $\tan\delta''$ and the amplitude of torsional oscillations θ_0 are given by:

$$\tan\delta''=\frac{\omega R+k'}{k\cdot I\omega^2\cdot\omega X}, \quad (9)$$

and

$$\theta_0=\frac{M_0}{(\omega R+k')\sqrt{1+1/\tan^2\delta''}}. \quad (10)$$

Thus the damping torque imposed by the soft matter $R+iX$ is obtained from Eqs. (9) and (10):

$$R=\frac{M_0}{\omega\theta_0\sqrt{1+1/\tan^2\delta''}}-\frac{k'}{\omega}, \quad (11)$$

and

$$X=\frac{k-I\omega^2}{\omega}-\frac{M_0}{\omega\theta_0\sqrt{1+1/\tan^2\delta''}}. \quad (12)$$

Let us use the $G^*=G'+iG''$ to represent dynamic shear viscoelasticity of soft matter. The elastic term G' and viscous term G'' are given by [9]:

$$G'=\frac{1}{4S^2r^4}\frac{R^2-X^2}{\rho}, \quad (13)$$

and

$$G''=\frac{1}{4S^2r^4}\frac{2RX}{\rho}. \quad (14)$$

To conclude, we present the basic concept of torsion pendulum-based mechanical spectrometer to characterize soft matter. It can be safely anticipated that the pendulum-based mechanical spectrometer will become an important instrument to study soft matter.

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