DOI: 10.24425/118910

W. WOŁCZYŃSKI*[#]

DEVELOPMENT OF THE JACKSON AND HUNT THEORY FOR RAPID EUTECTIC GROWTH

A development of the Jackson-Hunt's theory is delivered. Contrary to Jackson-Hunt's theory for ideally coupled growth the current description is dealing with the coupled eutectic growth which is more realistic than an ideal course of eutectic structure formation. Thus, the undercooling of every eutectic phase is not equal to each other. A new boundary condition is introduced to solve the diffusion equation. According to this condition, the eutectic concentration is always maintained at the triple point of the solid / liquid (s/l) interface. Therefore, the solution to diffusion equation is given separately for both lamellae. The mass balance is satisfied by the current solution. Both thermodynamic equilibrium and mechanical equilibrium are assumed to be situated at the triple point of the s/l interface, only. A protrusion of the leading phase over the wetting phase is defined mathematically due to the mass balance fulfilment. The current description is associated with the asymmetrical phase diagrams. Finally, the current description is applied to interpretation of the rapid eutectic growth. Therefore, Aziz's concept for the changes of partition ratio versus growth rate is introduced into the description. As a result, the rapid formation of the eutectic structure is described by the oscillatory mode. Interpretation of the oscillatory mode of the eutectic structure formation is illustrated in the arbitrary eutectic phase diagram. The eutectic structure, obtained through the detonation gas spraying onto the steel substrate (rapid solidification) is delivered to illustrate the present description.

Keywords: Jackson and Hunt theory, diffusion equation; coupled eutectic growth, Aziz's theory, oscillatory mode of eutectic growth

1. Introduction

Many experiments involve the rapid solidification. Particularly, the detonation method (D-Gun spraying onto the steel substrate) applied to creation of some intermetallic compounds / phases and solid solutions accompanied by the eutectic precipitates promotes the rapid solidification.

1a. Rapidly formed eutectic structure. Experiments

Some sub-layers can evince even the amorphous morphology, [1,2]. The partially melted, then rapidly solidified Fe-Al particles form multi-layers morphology which can appear in sequence, [3]. Sometimes, the Ni-Al – transition interlayer is added to improve the adherence of the Fe-Al – coating to the steel substrate, [4]. Solidification of the partially melted Fe-Al particles sprayed onto the steel substrate leads to formation of a kind of micro-joints in the coating. In some situations, the rapidly formed eutectic structure is observed, Fig. 1.

It should be emphasized that the oscillatory mode of some sub-layers formation was revealed / detected, Fig. 2, and described / illustrated, Fig. 3, [5].

The eutectic precipitates were accompanied the cellular morphology created due to the unidirectional solidification during the DGS experiment, Fig. 4.

Usually, some parts of the Fe-Al particles are melted and then solidified rapidly but some parts of these particles remain non-melted, [6]. The current observations of the rapidly formed morphology confirm the presence of rapidly solidified eutectic in the neighborhood of oriented cells, Fig. 4.

Therefore, the Jackson and Hunt theory, [7], formulated for the formation of the micro-field of solute concentration is subjected to a modification in order to describe the mechanism of the rapid solidification of the eutectic phases.

1b. Some modifications and applications of the Jackson and Hunt theory

The Jackson and Hunt theory is dealing with the so-called ideally coupled growth of the eutectics. Solving the diffusion equation for the stationary eutectic growth, the authors, [7], were able to predict the solute concentration micro-field ahead of the moving s/l interface. Unfortunately, the solution was obtained by imposing the common cosine function for both lamellae. Therefore, the mass balance is violated in this description of the

* INSTITUTE OF METALLURGY AND MATERIALS SCIENCE, POLISH ACADEMY OF SCIENCES, 25 REYMONTA STR., 30 059 KRAKÓW, POLAND

[#] Corresponding author: w.wolczynski@imim.pl

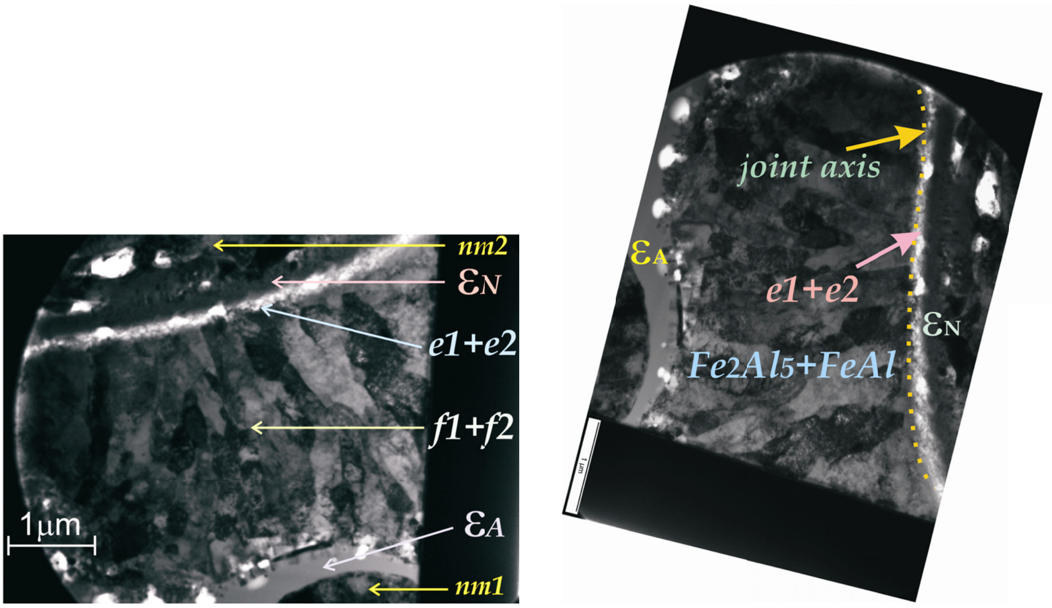


Fig. 1. Micro-joints revealed in the Fe-Al particle partially melted during its D-Gun deposition onto the steel substrate, a/ general view on the particle morphology, $nm1$; $nm2$ – non-melted parts of the particle forming a joint; ε_N – non-equilibrium form of the ε – phase; $f1 + f2$ two different phase formed through an oscillatory mode; ε_A – amorphous form of the ε – phase; $e1 + e2$ – eutectic precipitates, b/ the same morphology with the dotted line illustrating the joint axis

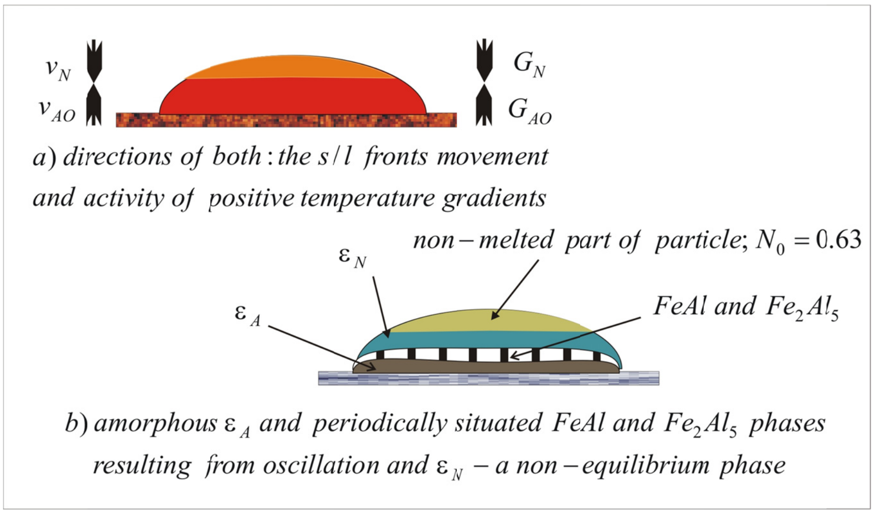


Fig. 2. Schematic view on the micro-joint formed from the melted part of the Fe-63Al particle deposited on the steel substrate by the DGS-method (with no eutectic precipitates)

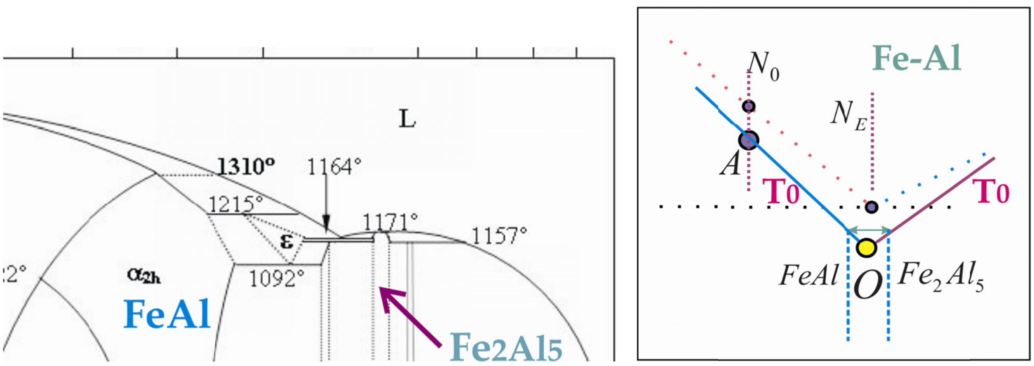


Fig. 3. Thermodynamic model for an ideal formation of the Fe_2Al_5, and FeAl – phases in the micro-joint, a/ phase diagram for stable equilibrium, b/ schematically shown phase diagram for meta-stable equilibrium with the T_0 – temperature; A – hypothetical point for the ε_A – amorphous phase formation with $k = 1$ (partition ratio); N_0 – nominal solute concentration of the particle; the ε – phase field disappearing is visible; the FeAl, and Fe_2Al_5 phases are in the neighborhood of the O – point of oscillation at which both mentioned phases are formed

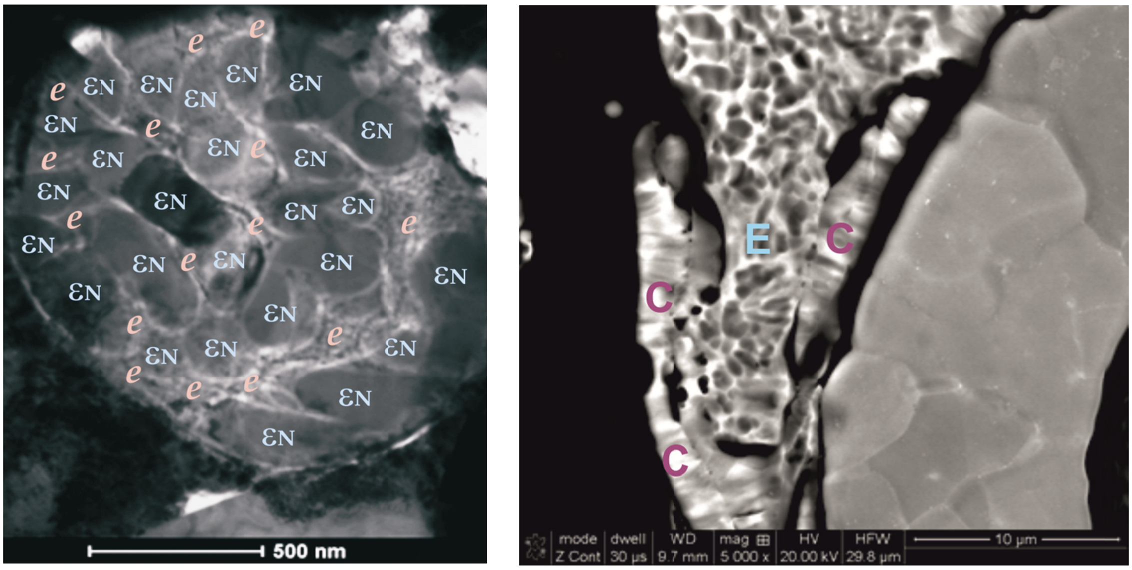


Fig. 4. Morphology of the Fe-Al coating deposited on the steel substrate by the D-Gun method, (coating about 100 mm thick formed from the Fe-Al powder containing particles of 40-75 mm in diameter), a/ cells of the ε_N – phase surrounded by the eutectic precipitates, b/ *E* – an eutectic formed during rapid solidification; *C* – columnar morphology localized in the vicinity of the eutectic precipitates and growing in two different directions

concentration micro-field. As a result, the connected temperature micro-field suggest that a part of the s/l interface of one lamella could be melted, [8]. It occurs since this part of the s/l interface is constitutionally overheated instead of be undercooled.

Some further modifications of the Jackson and Hunt theory consider the stability of the eutectic growth with the fulfilment of mechanical equilibrium at the triple point of the s/l interface, [9]. Although the stationary state is ensured some faults can appear in the lamellar eutectic structure, [10].

According to the Jackson and Hunt theory the spacing of lamellar structure is controlled / selected by the minimum undercooling condition. This condition is equivalent to the condition of maximum velocity of the s/l interface movement, [11]. Unfortunately, both conditions are the intuitive criteria, only. In reality, the eutectic growth occurs near the minimum, [7,8].

The effect of capillarity parameters (mechanical equilibrium) is essential for the stationary eutectic growth, [12]. Also the forced convection results in some disturbances of lamellar eutectic structure because the solute concentration micro-field is perturbed by the mentioned phenomenon, [13]. The fault-free lamellar structure seems to be obtainable when the eutectic thin films are solidified, [14]. The wetting phenomena can also be referred to the eutectic structure formation and finally to the Jackson and Hunt theory, [15].

Even the appearance of the tilt domains within the generally stationary lamellar structure can be connected theoretically with the Jackson and Hunt theory, [16]. It was proved that the description of the so-called quasi-stationary eutectic growth originates from the Jackson and Hunt theory, [17].

The mentioned models / treatments based on the Jackson and Hunt theory assume the ideally coupled growth of the lamellar or rod-like structure ($\Delta T_\alpha = \Delta T_\beta$) and fulfilment of the minimum undercooling condition. However, it was proved theoretically, that the regular eutectic growth is located within a certain interval of spacing selection near the minimum, [18].

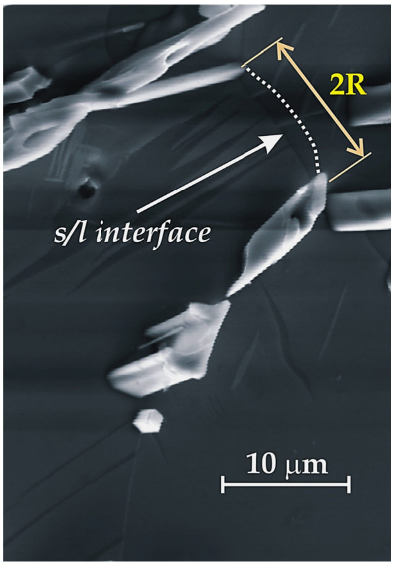
$\Delta T_S; S = \alpha, \beta$ is the s/l interface undercooling of a given eutectic phase.

Some similarities between the description of the cellular – eutectic growth occurring along the eutectic mono-variant line towards the three phases region and Jackson and Hunt theory are perceptible, [19].

The Jackson and Hunt theory was very fruitful in the development of the concept of eutectic structure transformation (lamellar into rod-like), [20]. However, in this case, the criterion of minimum entropy production justified mathematically (by the *Liouville's* theorem) was applied instead of the intuitive condition of minimum undercooling.

1c. Role of the Jackson and Hunt theory in the description of irregular eutectic growth

An unusual significance of the famous Jackson and Hunt theory has been emphasized by its application to all known descriptions of irregular eutectic growth. For example, the theory of branched limited growth of irregular eutectics assumes the localization of the regular eutectic structure formation (regular structure appearing locally among generally irregular morphologies) exactly at the undercooling minimum where the Jackson and Hunt theory is applicable, [21]. Thus, the regular eutectic spacing can be treated as the minimal distance visible among all distances revealed within the generally irregular eutectic morphology, [22-25]. Two extremely different spacing: minimal one (regular spacing existing among the generally irregular morphologies) and maximal one with extremely perturbed non-faceted eutectic phase are shown in Fig. 5.



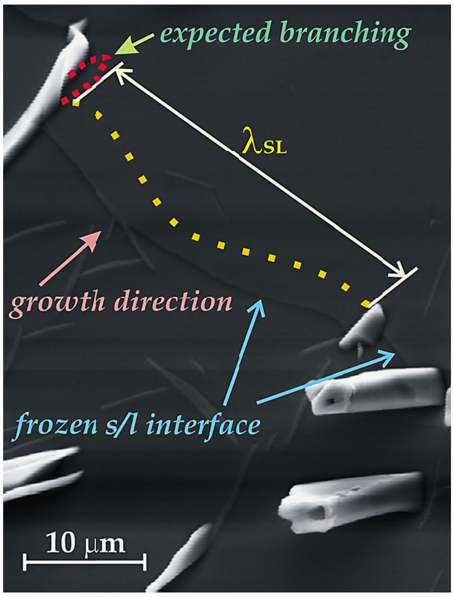


Fig. 5. Shapes of the non-faceted phase (s/l interface as frozen) revealed for stationary growth of the rod-like irregular eutectic (with branching phenomenon), a/ parabolic shape of the s/l interface (of the non-faceted phase) typical for regular eutectic existing among the generally irregular eutectic morphologies, b/ marginally stable s/l interface of irregular eutectic (of the non-faceted phase) typical for maximum perturbation wave appearing at this interface just when the morphological bifurcation of the faceted eutectic phase is expected

2. Current development of the Jackson and Hunt theory

Some modifications of the Jackson and Hunt theory tried to improve the understanding of physical mechanisms of solidification. For example, a simple solution is delivered for the nearly isothermal s/l interface which differs from that given by the Jackson and Hunt theory only by a corrective factor reflecting the density differences between eutectic phases, [26]. Based on this modification, it can be concluded that the real shape of the eutectic phases is to be parabolic one for regular structure formation as shown in Fig. 5. Thus, two types of the shape of the regular eutectic s/l interface are to be differentiated: parabolic convex and parabolic concave, Fig. 6.

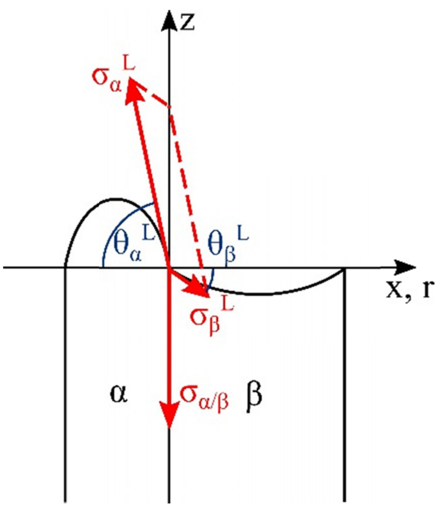
A modified relationships between growth rate, lamellar spacing and the s/l interface undercooling are the subject of recent generalization of the Jackson and Hunt theory, [27].

According to some suggestions delivered for the rapid eutectic growth, [23], a limit of absolute stability can be reached. However, at first, the ideally coupled eutectic growth assumed by the Jackson and Hunt theory is to be replaced by the coupled eutectic growth, [28]. Thus,

$$\Delta T_\alpha \neq \Delta T_\beta \tag{1}$$

as required by the asymmetrical eutectic phase diagrams. Consequently,

$$C_0^\alpha\left(S_\alpha,0\right) = C_S^\alpha\left(S_\alpha,0\right) - C_E < 0 \tag{2a}$$



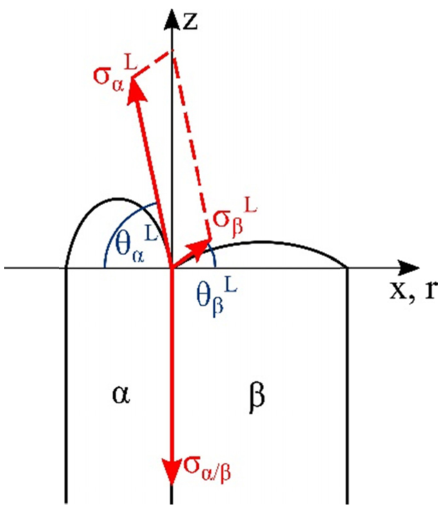


Fig. 6. Mechanical equilibrium at the triple point of the s/l interface for stationary growth of the rod-like / lamellar eutectic structure, a/ for the convex / concave interface, b/ for the convex / convex interface

$$C_0^\beta(S_\alpha,0)=C_S^\beta(S_\alpha,0)-C_E>0 \tag{2b}$$

C_S^α – equilibrium solute concentration in the α – phase,

C_S^β – equilibrium solute concentration in the β – phase,

C_E – solute concentration at the eutectic point.

The diffusion equation for the stationary eutectic growth is as follows:

$$\frac{\partial^2 \delta C}{\partial x^2}+\frac{\partial^2 \delta C}{\partial z^2}+\frac{v}{D}\frac{\partial \delta C}{\partial z}=0 \tag{3}$$

C – the B – solute concentration in the liquid.

The proposed solution to Eq. (3) can be given as a certain product:

$$\delta C(x,z)=X(x)\,Z(z) \tag{4}$$

with the co-ordinate system attached to the plane s/l interface just at the middle of the α – phase lamella, Fig. 7.

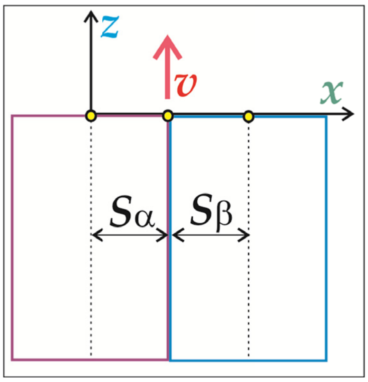


Fig. 7. Coordinate system attached at the s/l interface moving with the v – velocity, and definitions of the half the width of both eutectic lamellae, S_α, S_β, respectively.

Quite new boundary condition is inserted to the current description:

$$\delta C(S_\alpha,z)=C(S_\alpha,z)-C_E=0 \tag{5}$$

The above boundary condition simplifies to formulate the solution, Eq. (4), separately for both lamellae.

The Jackson and Hunt theory does not take into account Eq. (5). However, this very important and fruitful in consequences assumption given by Eq. (5) does not allow to appear a discontinuity of solute undercooling (constitutional undercooling) and resulting micro-filed of the temperature at the s/l interface. Thus,

$$Z(z)=\exp\left[\left(-\frac{v}{2D}-\sqrt{\frac{v^2}{4D^2}+\omega^2}\right)z\right] \tag{6}$$

$$X(x)=A\cos(\omega x)+B\sin(\omega x) \tag{7}$$

D – diffusion coefficient in the liquid.

The other boundary conditions are similar to those used in the Jackson and Hunt theory:

$$\frac{\partial C}{\partial x}=0,\ \text{for}\ x=0,\ \text{and for}\ x=S_\alpha+S_\beta \tag{8}$$

$$\frac{\partial C}{\partial z}=\mp\frac{vC_0^S}{D},\ S=\alpha,\beta\ \text{for}\ z=0,\quad \text{and}\ 0\le x<S_\alpha\quad S_\alpha<x\le S_\alpha+S_\beta \tag{9}$$

After some rearrangements and with the application of plane s/l interface, Fig. 7 the description of the solute micro-field ahead of the s/l interface is:

A) for the α – eutectic phase, that is for $x\in[0,S_\alpha]$, $z\ge 0$.

The value of the B, and ω – parameters results from the application of the following (modified) condition:

$$\left.\frac{\partial \delta C(x,z)}{\partial x}\right|_{x=0}=0 \tag{8a}$$

So, the formula: $-\omega A\sin(\omega\cdot 0)+\omega B\cos(\omega\cdot 0)=0$, yields $B=0$. Additionally,

$$\omega=\omega_{2n-1}=\frac{(2n-1)\pi}{2S_\alpha},\ n=1,2,\ldots$$

Combining Eq. (4), Eq. (6), and Eq. (7) with Eq. (5) and Eq. (8a)) the solution to Eq. (3) is:

$$\delta C(x,z)=\sum_{n=1}^{\infty}A_{2n-1}\cos\left(\frac{(2n-1)\pi x}{2S_\alpha}\right)\exp\left[\left(-\frac{v}{2D}-\sqrt{\frac{v^2}{4D^2}+\left(\frac{(2n-1)\pi}{2S_\alpha}\right)^2}\right)z\right] \tag{10}$$

where, A_{2n-1}, are constant. The following conditions allow to define the A_{2n-1} – parameter:

$$\left.\frac{\partial \delta C(x,z)}{\partial z}\right|_{z=0}=f_\alpha(x);\ f_\alpha(x)<0,\ x\in[0,S_\alpha] \tag{11}$$

then:

$$\left.\frac{\partial \delta C(x,z)}{\partial z}\right|_{z=0}=\sum_{n=1}^{\infty}\left(A_{2n-1}\left(-\frac{v}{2D}-\sqrt{\frac{v^2}{4D^2}+\left(\frac{(2n-1)\pi}{2S_\alpha}\right)^2}\right)\cos\left(\frac{(2n-1)\pi x}{2S_\alpha}\right)\right) \tag{12}$$

It is worth to note that the introduced function $f(x)$, has the following properties:

$$f(x),\ -2S_\alpha\le x\le 2S_\alpha,\quad f(-x)=f(x),\ f(x+2S_\alpha)=-f(x) \tag{13}$$

$$f(x)\approx\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\left(\frac{n\pi x}{2S_\alpha}\right) \tag{14}$$

$$a_n = \frac{1}{S_\alpha}\int_0^{2S_\alpha} f(x)\cos\left(\frac{n\pi x}{2S_\alpha}\right)dx \tag{15}$$

Assuming that: $f(x + 2S_\alpha) = -f(x)$, it yields: $a_{2k} = 0$, $k = 0,1,2,...$ for $n = 2k$.

$$a_{2k-1} = \frac{2}{S_\alpha}\int_0^{S_\alpha} f(x)\cos\left(\frac{(2k-1)\pi x}{2S_\alpha}\right)dx,$$
$$k = 1, 2, \ldots \quad \text{for} \quad n = 2k-1,\ k = 1,2,\ldots$$

Finally, the *Fourier's* series for $f(x)$ can be shown as:

$$f(x) \approx \sum_{k=1}^{\infty} a_{2k-1}\cos\left(\frac{(2k-1)\pi x}{2S_\alpha}\right) \tag{16}$$

Finally, the definition the A_{2n-1} – parameter is:

$$A_{2n-1} = \left(-\frac{v}{2D} - \sqrt{\frac{v^2}{4D^2} + \left(\frac{(2n-1)\pi}{2S_\alpha}\right)^2}\right)^{-1} \frac{2}{S_\alpha}\int_0^{S_\alpha} f_\alpha(x)\cos\left(\frac{(2n-1)\pi x}{2S_\alpha}\right)dx \tag{17}$$

It can be readily proved that the obtained solution, Eq. (10), and Eq. (17), satisfies the conditions applied to the current description:

$$\left.\frac{\partial\delta C(x,z)}{\partial x}\right|_{x=0} = \left.\frac{\partial\delta C(x,z)}{\partial x}\right|_{x=2S_\alpha} = 0 \tag{18}$$

$$\left.\frac{\partial\delta C(x,z)}{\partial z}\right|_{z=0} = f_\alpha(x) = f_\alpha(-x) = -f_\alpha(-x+2S_\alpha) = -\left.\frac{\partial\delta C(-x+2S_\alpha,z)}{\partial z}\right|_{z=0} \tag{19}$$

according to the assumptions:

$$f_\alpha(-x) = f_\alpha(x),\ f_\alpha(x + 2S\alpha) = -f_\alpha(x)$$

B/ analogously, for the β – eutectic phase, that is for:

$$x \in [S_\alpha, S_\alpha + S_\beta],\ z \geq 0:$$

$$\delta C(x,z) = \sum_{n=1}^{\infty}\left(B_{2n-1}\cos\left(\frac{(2n-1)\pi(x - S_\alpha + S_\beta)}{2S_\beta}\right)\exp\left[\left(-\frac{v}{2D} - \sqrt{\frac{v^2}{4D^2} + \left(\frac{(2n-1)\pi}{2S_\beta}\right)^2}\right)z\right]\right) \tag{20}$$

$$B_{2n-1} = \left(-\frac{v}{2D} - \sqrt{\frac{v^2}{4D^2} + \left(\frac{(2n+1)\pi}{2S_\beta}\right)^2}\right)^{-1} \frac{2}{S_\beta}\int_{S_\alpha - S_\beta}^{S_\alpha} f_\beta(x)\cos\left(\frac{(2n+1)\pi(x - S_\alpha + S_\beta)}{2S_\beta}\right)dx \tag{21}$$

with,

$$\left.\frac{\partial\delta C(x,z)}{\partial z}\right|_{z=0} = f_\beta(x), \quad x \in [0, S_\beta] \tag{22}$$

Both, separately obtained solutions to Eq. (3), for the α – phase lamella, (Eq. (10) with the accompanying Eq. (17)) and for the β – phase lamella, (Eq. (20) with the accompanying Eq. (21)) are presented schematically in Fig. 8.

The concept of the coupled eutectic growth, Eq. (1), is taken into account, Fig. 8.

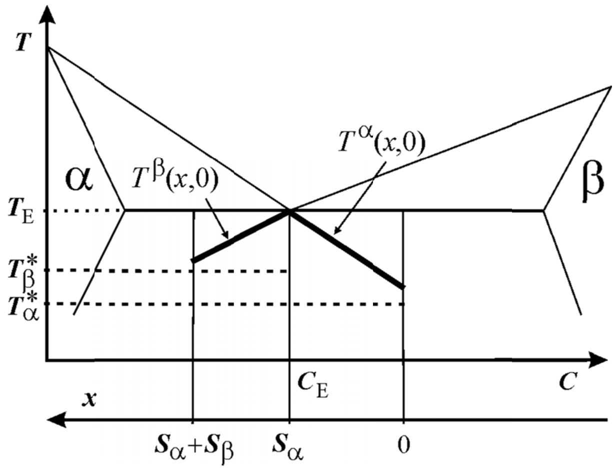


Fig. 8. Solute temperature / concentration micro-field (bold lines) localized in the arbitrary phase diagram for $z = 0$, according to the concept of the coupled eutectic growth, the x – geometrical axis showing the lamellae size is added

Consequentially, these solutions, can also be shown in the both, $C(x)|_{z=0}$, and $T(x)|_{z=0}$ co-ordinate systems, Fig. 9.

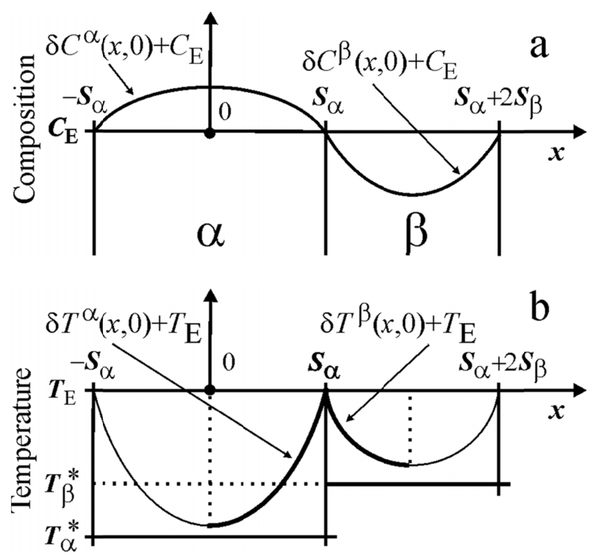


Fig. 9. Localization of the thermodynamic equilibrium, (C_E, T_E), in frame of the results of the current description for the lamellar eutectic structure formation, a/ solute concentration micro-field with the localization of the C_E – parameter, b/ solute temperature (equilibrium temperature) micro-field with the localization of the T_E – parameter, T^*_α, T^*_β – temperature of the s/l interface for the α – phase lamella, and for the β – phase lamella (curvature undercooling is taken into account), respectively

Concluding remarks

The obtained solution is to be verified by means of the so-called local mass balance calculation:

$$\int_{0}^{S_\alpha} \delta C(x,0)\,dx + \int_{S_\alpha}^{S_\alpha+S_\beta} \delta C(x,d)\,dx = 0 \qquad (23)$$

The local mass balance, Eq. (23), is satisfied under condition that the d – protrusion of the β – leading eutectic phase over the α – wetting eutectic phase is taken into account, theoretically, Fig. 10. It is justified since the phase protrusion has been observed experimentally, [29,28]. Moreover, the protrusion is to be expected according to the assumption of coupled eutectic growth, $\Delta T_\alpha \neq \Delta T_\beta$, Eq. (1).

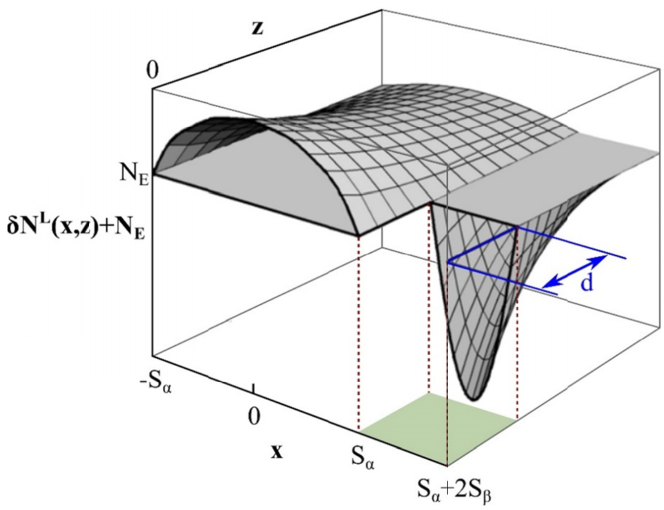


Fig. 10. Visualization of the local mass balance with the phase protrusion taken into account

Eq. (23) yields:

$$\sum_{n=1}^{\infty} A_{2n-1} \frac{2S_\alpha(-1)^{n-1}}{(2n-1)\pi} - \sum_{n=1}^{\infty} B_{2n-1} \frac{2S_\beta(-1)^{n-1}}{(2n-1)\pi} \exp\left(-\frac{vS_\beta + \sqrt{v^2 S_\beta^2 + (2n-1)^2 D^2 \pi^2}}{2DS_\beta} d \right) = 0 \qquad (24)$$

The above mass balance, Eq. (24), allows to formulate the theoretical definition for the d – phase protrusion, Fig. 10. The prediction of the d – phase protrusion is the advantage of the current description for the stationary eutectic growth. The resultant definition of the d – phase protrusion is given as follows:

$$\sum_{n=1}^{\infty} A_{2n-1} \frac{(-1)^{n-1}}{(2n-1)} \left(1 - \frac{S_\alpha \left(vS_\beta + \sqrt{v^2 S_\beta^2 + (2n-1)^2 D^2 \pi^2} \right)}{S_\beta \left(vS_\alpha + \sqrt{v^2 S_\alpha^2 + (2n-1)^2 D^2 \pi^2} \right)} \exp\left(-\frac{vS_\beta + \sqrt{v^2 S_\beta^2 + (2n-1)^2 D^2 \pi^2}}{2DS_\beta} d \right) \right) = 0 \qquad (25)$$

The current description of the solute concentration micro-field, Eq. (10), and Eq. (20), can be perfectly, mathematically reduced to analogous description delivered by Jackson and Hunt theory. However, it must be assumed that $S_\alpha = S_\beta$. In the consequence, $C_0^\alpha = C_0^\beta$ according to the condition: $B_0 = 0$ (the B_0 – parameter is defined in the Jackson and Hunt theory). Moreover, the d – phase protrusion disappears, ($d \to 0$), when $S_\alpha = S_\beta$, and $C_0^\alpha = C_0^\beta$ in the current description. Thus, the present coupled eutectic growth could be reduced to the ideally coupled eutectic growth assumed by the Jackson and Hunt theory.

Consequentially, the C_E – eutectic concentration comes back to the α/β – phase boundary, that is to the triple point of the s/l interface in the Jackson and Hunt theory (when $S_\alpha = S_\beta$). In this case, the local mass balance is satisfied (even for $z = 0$) in the Jackson and Hunt theory. Then, the Jackson and Hunt theory accepts the condition defined by Eq. (5), when $S_\alpha = S_\beta$.

It can be concluded that the current description is a general formulation for the solute concentration micro-field and can be applied to all asymmetrical phase diagrams for which, $S_\alpha \neq S_\beta$; $C_0^\alpha \neq C_0^\beta$. If so, it is possible to modify the current equations in order to understand the mechanism of the rapid eutectic growth frequently observed experimentally, [1-6]. Contrary to the known suggestions, [23], according to which the numerical treatment of the problem is possible, only, the current description tries to analyze the rapid eutectic growth analytically.

Thus, the concept of varying partition ratio versus solidification rate, [30], is to be applied to the current description as shown schematically in Fig. 3b. In this case an oscillatory mode of the eutectic structure formation could be expected. The mentioned mode differs from that which appears during the slow growth of the single crystal reinforced by the eutectic precipitates, [31,33]. The current illustration of the oscillatory mode of the eutectic structure formation is shown schematically in Fig. 11 for the k – partition ratio equal to unity.

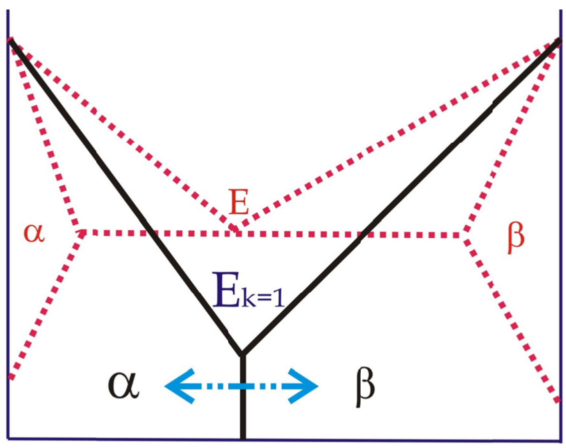


Fig. 11. Arbitrary eutectic phase diagram (red dashed lines) and modified phase diagram (bold lines) for the rapid eutectic growth with $k = 1$, and $C_0^\alpha = 0$; $C_0^\beta = 0$; oscillatory mode of the both eutectic phases appearance is illustrated by double arrows

Some equations of the current descriptions are to be modified while considering the rapid eutectic growth. First, $f_\alpha(x) = 0$,

$x \in [0, S_\alpha]$, Eq. (11), and $f_\beta(x) = 0$, $x \in [S_\alpha, S_\alpha + S_\beta]$, Eq. (22). In simplification, both functions: $f_\alpha(x); f_\beta(x)$ can be replaced by the condition applied in the Jackson and Hunt theory. Then, the following conditions arise: $f_\alpha(x) = \dfrac{vC_0^\alpha}{D} = 0$; $f_\beta(x) = \dfrac{vC_0^\beta}{D} = 0$ according to the eutectic phase diagram for rapid solidification, Fig. 11, where $C_0^\alpha = 0$; $C_0^\beta = 0$.

Consequentially, $A_{2n-1} = 0$; $B_{2n-1} = 0$, Eq. (17), and Eq. (21). It results in disappearing of the solute concentration micro-field ahead of the s/l interface, $\delta C(x,z) \to 0$, Eq. (10), and Eq. (20). Instead, an oscillatory mode of the eutectic structure formation appears, Fig. 11.

The current development of the Jackson and Hunt theory (worked out for the lamellar eutectic structure formation, only) shows that the triple point of the s/l interface plays an unusual / essential role in the eutectic structure formation. Not only the mechanical equilibrium is situated at this point, Fig. 6, but the thermodynamic equilibrium as well, Eq. (5), Fig. 8, and Fig. 9.

The localization of the thermodynamic equilibrium in the considered systems is necessary for the calculation of the entropy production per unit time, [32], and its minimization required by the stationary state definition. Usually, the thermodynamic equilibrium was attributed to the whole s/l interface, [32]. The current development of the Jackson and Hunt theory indicates that the thermodynamic equilibrium can be attributed to the α/β – phase boundary and particularly to its moving triple point.

Acknowledgements

The financial support was provided by the National Science Center (NCN), Poland under grant No. 2015/19/B/ST8/02000.

REFERENCES

[1] A. Pawłowski, T. Czeppe, Ł. Major, C. Senderowski, Archives of Metallurgy and Materials **54**, 783-788 (2009).

[2] C. Senderowski, A. Pawłowski, Z. Bojar, W. Wołczyński, M. Faryna, J. Morgiel, Ł. Major, Archives of Metallurgy and Materials **55**, 373-381 (2010).

[3] C. Senderowski, Z. Bojar, W. Wołczyński, A. Pawłowski, Intermetallics **18**, 1405-1409 (2010).

[4] A. Pawłowski, C. Senderowski, W. Wołczyński, J. Morgiel, Ł. Major, Archives of Metallurgy and Materials **56**, 71-79 (2011).

[5] W. Wołczyński, C. Senderowski, J. Morgiel, G. Garzeł, Archives of Metallurgy and Materials **59**, 209-217 (2014).

[6] C. Senderowski, Journal of Thermal Spray Technology **23**, 1124-1134 (2014).

[7] K.A. Jackson, J.D. Hunt, Transactions of the Metallurgical Society of AIME **236**, 1129-1142 (1966).

[8] J.D. Hunt, K.A. Jackson, Transactions of the Metallurgical Society of AIME **236**, 843-852 (1966).

[9] D.T.J. Hurle, E. Jakeman, Journal of Crystal Growth **3/4**, 574-582 (1968).

[10] J.E. Gruzleski, W.C. Winegard, Journal of the Institute of Metals **96**, 301-303 (1968).

[11] G. Bolze, M.P. Puls, J.S. Kirkaldy, Acta Metallurgica **20**, 73-85 (1972).

[12] S. Budurov, S. Yaneva, N. Stoichev, B. Phan Van, Kristall und Technik **10**, 505-510 (1975).

[13] J.M. Quenisset, R. Naslain, Journal of Crystal Growth **54**, 465-474 (1981).

[14] H.E. Cline, Metallurgical Transactions **15A**, 1013-1017 (1984).

[15] A. Mortensen, Materials Science and Engineering **A135**, 1-11 (1991).

[16] G. Faivre, J. Mergy, Physical Review **46A**, 963-972 (1992).

[17] K. Kassner C. Misbah, R. Baumann, Physical Review **51E**, 2751-2754 (1995).

[18] M. Dong, J. Wanqi, Proceedings of the 4th Decennial International Conference on Solidification Processing, Sheffield 1997, 394-396.

[19] T. Himemiya, Science and Technology of Advanced Materials **2**, 325-329 (2001).

[20] W. Wołczyński, International Journal of Thermodynamics **13**, 35-42 (2010).

[21] D.J. Fisher, W. Kurz, Acta Metallurgica **28**, 777-794 (1980).

[22] H. Jones, W. Kurz, Zeitschrift fur Metallkunde **72**, 792-797 (1981).

[23] W. Kurz, R. Trivedi, Acta Metallurgica and Materialia **38**, 1-17 (1990).

[24] P. Magnin, J. T. Mason, R. Trivedi, Acta Metallurgica and Materialia **39**, 469-480 (1991).

[25] E. Guzik, D. Kopyciński, Metallurgical and Materials Transactions **37A**, 3057-3067 (2006).

[26] P. Magnin, R. Trivedi, Acta Metallurgica and Materialia **39**, 453-467 (1991).

[27] A. Ludwig, S. Leibbrandt, Materials Science and Engineering **A 375-377**, 540-546 (2004).

[28] W. Wołczyński, Defect and Diffusion Forum **272**, 123-138 (2007).

[29] V.L. Davies, Journal of the Institute of Metals **93**, 10-16 (1964-1965).

[30] M.J. Aziz, Journal of Applied Physics **53**, 1158-1168 (1982).

[31] G. Boczkal, B. Mikułowski, W. Wołczyński, Materials Science Forum **649**, 113-118 (2010).

[32] G. Lesoult, M. Turpin, Memoires Scientifiques de la Revue de Metallurgie, **66**, 619-631 (1969).

[33] G. Boczkal, Arch. Met. Mater. **58**, 1019-1022 (2013).